Analysis of the $K^+ \to e^+ \nu \gamma$ decay

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Abstract
A study of the kaon leptonic radiative $K^+ \to e^+ \nu_e \gamma (SD^+)$ decay with a partial sample of the data collected in 2007 is reported. The signal event selection, some preliminary trigger studies and the analysis strategy for the measurement of the decay rate and FFs are presented. A detailed discussion of systematic uncertainties and the final result is foreseen in a subsequent note.

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1 The $K^+ \to e^+ \nu_e \gamma$ decay

1.1 Introduction and theoretical framework

The $K^+ \to l^+ \nu_l \gamma$ ($l = e, \mu$) process receives two contributions to the total decay amplitude: “Inner Bremsstrahlung” IB, “Structure-Dependent” SD and a term of interference INT between these two components [1,2]. The IB part is purely electromagnetic and to the lowest order can be predicted on general grounds (Low theorem). The SD part receives electro-weak and hadronic contributions and is sensitive to the kaon structure itself, in particular for what concerns its effective coupling to the photon. This coupling can proceed through vector and axial-vector effective currents that can be parametrized in terms of vector and axial-vector Form Factors ($F_V(p^2)$, $F_A(p^2)$), where $p^2 = (p_K - p_\gamma)^2$ is the momentum transferred to the leptonic pair $l\nu$. Terms of given helicity\(^1\) (SD\(^+\), sensitive to $F_V+F_A$ couplings) or (SD\(^-\), sensitive to $F_V-F_A$ couplings) may be disentangled from kinematical analysis and may lead to a deeper understanding of the kaon structure. Their corresponding interference terms with IB are INT\(^+\) and INT\(^-\). Predictions for $F_V(p^2)$ and $F_A(p^2)$ exist from low-energy systematic operator expansion (Chiral perturbation - ChPT theory, up to the $p^6$ order) and from specific models (Light front quark model, ChPT with vector-meson dominance or constituent quark model). From the theoretical point of view, the importance of this analysis on the $K^+ \to e^+ \nu_e \gamma$ decay ($K_{e2\gamma}$) is that it allows a quantitative comparison of these predictions.

\(^1\)The sign in SD\(^+\) and SD\(^-\) components is related to the polarisation of the radiative photon; there is no interfering term between these components.
In order to describe the kinematics of $K^+ \to e^+ \nu_e \gamma$ decay, two dimensionless Lorentz-invariant variables are introduced:

$$x = \frac{2p_K \cdot p_\gamma}{m_K^2}, \quad y = \frac{2p_K \cdot p_e}{m_K^2} \quad (1)$$

with $m_K$ the kaon mass, $p_K$ the kaon 4-momentum, $p_\gamma$ the photon 4-momentum and $p_e$ the electron 4-momentum. The variables $x$ and $y$ satisfy the following constraints:

$$0 \leq x \leq 1 - r_e, \quad 1 - x + \frac{r_e}{1 - x} \leq y \leq 1 + r_e \quad (2)$$

where $r_e = m_e^2/m_K^2$ and $m_e$ is the electron mass. In the kaon rest frame, $x$ ($y$) is proportional to the photon (electron) energy:

$$x = \frac{2E_\gamma}{m_K}, \quad y = \frac{2E_e}{m_K}. \quad (3)$$

The relation between the transferred momentum $p^2$ and $x$ is given by:

$$p^2 = m_K^2(1 - x).$$

Using the two kinematical variables of Eq. 3, the $x$ and $y$ scatter plots for the IB and SD± components are plotted in Fig. 1 and referred to as Dalitz plots in the rest of the note.

![Figure 1: Structure dependent at ChPT $O(p^6)$ and inner bremsstrahlung terms represented on Dalitz plots. SD+ (left), SD− × 10 (centre) and IB × 100 (right). The z axis represents the differential rate $d^2\Gamma/dxdy$ [GeV].](image)

The $K_{e2\gamma}$ differential decay rate for the structure dependent part is given by:

$$\frac{d^2\Gamma}{dxdy} (SD) = \frac{m_K^5|\alpha G_F|^2|V_{us}|^2}{64\pi^2} \times [(F_V + F_A)^2 f_{SD+}(x, y) + (F_V - F_A)^2 f_{SD-}(x, y)], \quad (4)$$

where $G_F$ is the Fermi constant and $V_{us}$ is the CKM matrix element. The form factors $f_{SD+}$ and $f_{SD-}$ represent the hadronic structure dependent contributions to the decay rate from SD+ and SD− channels, respectively, and they are expressed in terms of $x$ and $y$ as follow:
\[ f_{SD^+}(x, y) = (x + y - 1 - r_e) \cdot [(x + y - 1)(1 - x) - r_e], \]
\[ f_{SD^-}(x) = (1 - y + r_e) \cdot [(1 - x)(1 - y) + r_e]. \]

In Fig. 2 (left plot) the electron energy spectrum in the kaon rest frame for the favoured SD\(^+\) configuration [8] is showed. The photon is emitted preferentially anti-parallel to the electron. The contributions of SD (ChPT \(O(p^6)\)) and IB terms to the \(K_{e2\gamma}\) differential decay rate are also represented in Fig. 2 (right plot).

The ChPT at \(O(p^4)\) predicts constant form factors \(F_V(p^2) = F_V(0)\) and \(F_A(p^2) = F_A(0)\) [3]. The \(p^2\)-dependence of \(F_A(p^2)\) for the ChPT at \(O(p^6)\) is small and thus considered negligible, while \(F_V(p^2)\) exhibits a linear dependence on \(p^2\) (or equivalently on \(x\)) [4]:

\[ F_V(x) = F_V(0) \cdot [1 + \lambda(1 - x)] \] (5)

In the LFQM the \(x\) dependence is more complex [4]. The theoretical predictions for the form factors in the various models are summarized in Tab. 1 [4]:

<table>
<thead>
<tr>
<th>Model</th>
<th>(F_V(0))</th>
<th>(F_A(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(O(p^4))</td>
<td>0.0945</td>
<td>0.0425</td>
</tr>
<tr>
<td>(O(p^6))</td>
<td>0.082</td>
<td>0.034</td>
</tr>
<tr>
<td>LFQM</td>
<td>0.106</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Table 1: The \(K_{e2\gamma}\) form factors at \(p^2 = 0\) in the ChPT at \(O(p^4)\), \(O(p^6)\) and LFQM

It is worth mentioning that the theoretical errors on the form factors are not quoted in [4]; an estimate of the uncertainties in the framework of ChPT is reported in [5], where the errors were obtained by looking at the spread in results obtained by using different computations of the low-energy constants entering the chiral lagrangian\(^2\) (Tab. 2).

<table>
<thead>
<tr>
<th>Form factor</th>
<th>ChPT estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_V(0))</td>
<td>0.078 (\pm) 0.005</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>0.3 (\pm) 0.1</td>
</tr>
<tr>
<td>(F_V(0) + F_A(0))</td>
<td>0.112 (\pm) 0.005</td>
</tr>
<tr>
<td>(F_V(0) - F_A(0))</td>
<td>0.044 (\pm) 0.010</td>
</tr>
</tbody>
</table>

Table 2: Theoretical estimation of the form factors in the ChPT framework as reported in [5]. The error on \(F_A(0)\) is estimated to be at the percent level while the uncertainty on \(F_V(0) - F_A(0)\) is the difference between the central values at \(O(p^6)\) and \(O(p^4)\).

\(^2\)In ChPT, the hadronic contributions are parametrized by a number of parameters, the so-called low-energy constants. The quality of theoretical predictions hinges to a large extent on the available information on those constants.
Figure 2: (Left) Sketch of the electron energy spectrum in the kaon rest frame ($E_\gamma^*$) for the favoured SD+ configuration [8]. The photon is emitted preferentially anti-parallel to the electron. (Right) The contributions of SD (ChPT $O(p^6)$) and IB terms to the differential decay rate (logarithmic scale).

1.2 Experimental status

In this section the most recent results of the $K^+ \rightarrow e^+\nu_e\gamma$ Branching Ratio and Form Factors will be reported. The PDG 2010 [9] quotes:

$$BR(K^+ \rightarrow e^+\nu_e\gamma) = (9.4 \pm 0.4) \cdot 10^{-6}$$

in the kinematic region $10 \text{ MeV} < E_\gamma^* < 250 \text{ MeV}$ and $p_e^* > 200 \text{ MeV}$. This value was obtained by the KLOE collaboration using the DAFNE accelerator in Frascati. The measurement is based on the observation of 1484 $K^\pm \rightarrow e^\pm\nu_e\gamma$ candidates in the above kinematic region. The SD− and IB contributions account for about 2% and 1.3% respectively of the total rate. The form factor parameters have been obtained by fitting the measured $E_\gamma^*$ distribution with the theoretical differential decay width, assuming the vector form factor expansion at the first order of $x$ as in Eq. 5. In the fitting procedure $F_V(0) - F_A(0)$ is kept fixed at the expectation value from ChPT at $O(p^4)$ and the results are [10]:

$$F_V(0) + F_A(0) = 0.125 \pm 0.007_{\text{stat}} \pm 0.001_{\text{syst}},$$

$$\lambda = 0.38 \pm 0.20_{\text{stat}} \pm 0.02_{\text{syst}},$$

which confirms at $\sim 2\sigma$ the presence of a slope in the vector form factor $F_V$. The $K_{\mu2\gamma}$ extraction of the combination $F_V(0) + F_A(0)$ differs from the value achieved with $K_{e2\gamma}$ at a $3\sigma$ level [11]:

$$F_V(0) + F_A(0) = 0.165 \pm 0.007_{\text{stat}} \pm 0.011_{\text{syst}}.$$
The combination \( F_V(0) - F_A(0) \) was only loosely constrained by \( K_{l3\gamma} \) measurements and its best determination, \( F_V(0) - F_A(0) = 0.077 \pm 0.028 \) (consistent with ChPT), comes from \( K \to \mu\nu e^+ e^- \) [12]. Very recently the ISTRA+ collaboration has reported the study of the \( K^- \to \mu^- \nu \gamma \) decay [13, 14]. The measurement of the sign and the value of the difference \( F_V - F_A \) was performed assuming constant form factors as in ChPT at \( O(p^4) \) and observing the interference term between IB and SD components in a sample of about 22k \( K^- \to \mu^- \nu \gamma \) events. The ISTRA+ results [13]:

\[
F_V(0) - F_A(0) = 0.21 \pm 0.04_{\text{stat}} \pm 0.03_{\text{syst}}
\]

and [14]:

\[
F_V(0) - F_A(0) = 0.126 \pm 0.027_{\text{stat}} \pm 0.047_{\text{syst}}
\]

are respectively about 3 and 1.5 \( \sigma \) above the ChPT predictions [5]. The two results are referred to two alternative analysis of the ISTRA+ data.

2 \( K^+ \to e^+\nu_e\gamma(\text{SD}^+) \) event selection

Three analyses of the \( K^+ \to e^+\nu_e\gamma (\text{SD}^+) \) decay channel based on the data sample of period 5 (P5, runs 20410-20485) are on going. Data were collected in 2007 and the sample analysed in this note corresponds to about 40k of online \( K_{e2} \) decay events, which were acquired with a minimum bias trigger\(^3\) [6]. A common signal event selection has been established for the three analyses and all the following plots are obtained with the above data sample.

2.1 Charged track selection

The selection aims at identifying a well reconstructed track from the signal channel, while being independent as much as possible on the presence of possible tracks from accidental beam-induced activity. This selection is obtained in two steps:

- The definition of a “good” track is set as loose as possible to include with high efficiency the tracks from K decays, while minimising the probability to have an accidental track which is labelled as “good”.

- Once a “good” track is found, a tighter selection is applied to achieve higher quality tracks.

This is useful for the background rejection, as well as for restricting the signal sample to a given kinematic region and performing an accurate extraction of the form factors.

The selection of a charged track is mostly inherited from the \( K_{e2} \) analysis [7], it only differs for the selected fiducial decay region and track momentum (see \( Z_{\text{cutx}} \) and \( P_{\text{trk}} \) cuts). Within the first step of the track selection a sample of “bad” tracks is identified for each event; a

\(^3\)The \( K_{e2} \) main trigger.
The event is accepted only if exactly one “good” track is found: this is considered the lepton candidate track. Within the second step of the track selection the lepton candidate must satisfy the following additional requirements:

• positive charge $q > 0$.

• quality $> 0.7$ (quality = fraction of hits in each DCH view close in time to the average time of the total sample).

• Track impact points in the geometrical acceptance of the detectors:
  
  - track radius at the DCH 1,2,4 planes: $12 \text{ cm} < R_{DCCH} < 115 \text{ cm}$.
  - track radius at the CHOD plane: $14 \text{ cm} < R_{CHOD} < 115 \text{ cm}$.
  - track time given by the charged hodoscope$^4$: $118 \text{ ns} < t_{trk} < 158 \text{ ns}$.
  - distance to nearest dead cell at LKr $d > 2 \text{ cm}$.
  - octagonal shaped cut according to Mauro’s routine [15] with a margin parameter of 8 cm.

• $-1600 \text{ cm} < Z_{vtx} < 9000 \text{ cm}$ (Fig. 3).

• $CDA < 3 \text{ cm}$.

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$^4$Such variable is not filled for all events, when the charged hodoscope gives a value $\leq 0$ the time is taken from the DCH.
• $10 \text{ GeV}/c < P_{\text{trk}} < 55 \text{ GeV}/c$ (Fig. 4). The lower limit on the track momentum is chosen to satisfy the $E_{\text{LKr}} > 10 \text{ GeV}$ trigger condition (explained in detail in Sec. 4), together with the photon energy requirement it ensures more than 15 GeV energy deposit in the LKr. The upper value is mainly dictated by the $K^+ \rightarrow \pi^+\pi^0$ background and is chosen in order to minimize such contamination (see Fig. 5).

• The particle ID is based on the reconstruction of the ratio $E/p$ between the track energy deposition in the LKr calorimeter (associated cluster) and the track momentum as measured by the magnetic spectrometer [6]. The associated cluster is defined as the closest cluster to the track position at the LKr in a radius within 5 cm, in time $\Delta t_{\text{trk-ass.cl}} < 6 \text{ ns}$ and with its cells correctly reconstructed (cluster status variable < 4). The tracks with $0.95 < E/p < 1.1$ are identified as electrons (Fig. 6).

• A cut is applied on the track-associated cluster distance (Fig. 3) $d_{\text{trk-ass.cl.}} < 1.5 \text{ cm}$.

![Figure 3: (Left) Longitudinal decay vertex ($Z_{\text{vtx}}$) for data after the selection, except the $Z_{\text{vtx}}$ cut. (Right) Track-associated cluster distance ($d_{\text{trk-ass.cl.}}$) for data after the selection, except the $d_{\text{trk-ass.cl.}}$ cut. The arrows represent the applied cut.](image-url)
Figure 4: Track momentum distribution in the laboratory frame for data after the selection except the momentum cut: the peak at 65 GeV/c is due to $K^+ \rightarrow \pi^+\pi^0$ background events. The arrows represent the applied cut.

Figure 5: $M^2_{miss} = (p_K - p_e - p_\gamma)^2$ distribution vs the track momentum for $K^+ \rightarrow e^+\nu_e\gamma$ (left, signal) and $K^+ \rightarrow \pi^+\pi^0$ (right, bkg) MC events.
Figure 6: $E/p$ distribution for data: the depletion of events in the region $0.3 < E/p < 0.6$ is due to the L3 trigger (effect known from previous analyses).

2.2 Photon selection

The cluster associated to the photon from $K_{e2\gamma}$ decay is selected among the “good” clusters in the calorimeter. A good cluster satisfies each of the following criteria:

- not associated to any track.
- $E > 5$ GeV (Fig. 7). This requirement, together with the cut on the track momentum, ensures more than 15 GeV energy deposit in the LKr calorimeter, so that the trigger condition is fulfilled with high efficiency (see Sec. 4).
- within the LKr acceptance according to Mauro’s routine [15] with a margin parameter of 8 cm.
- in time with the lepton candidate track: $|\Delta t_{trk-cl}| < 6$ ns.
- distance from the nearest dead cell greater than 2 cm.
- cells of the cluster correctly reconstructed (cluster status variable < 4).
- distance from any other cluster $R_{cl-cl} > 20$ cm to avoid cluster overlapping and energy sharing effects.

The event is accepted only if exactly one good cluster has been found, this is considered the photon candidate cluster. Further cuts are applied on the candidate cluster:

- to suppress events in which the selected photon comes from the positron bremsstrahlung, a minimum distance of 8 cm between the photon cluster and the track extrapolation at the LKr plane, without considering the magnet bending, is required.
• If the selected photon interacts in the passive material traversed before reaching the LKr (e.g. the DCH flanges), the selected cluster may be one of its interaction products. In these cases, the corresponding reconstructed missing mass is shifted and contributes to the positive tail of the distribution. Such effect has been studied with a control sample of $K_{e3}$ decay. By using the photon trajectory slopes as introduced in Sec. 2.5, the data-MC comparison of the photon radius at the first DCH is represented in Fig. 8. This shows a discrepancy for photon radii $< 12$ cm, such distance being compatible with the actual size of the DCH flanges and thus indicating that the corresponding material budget in the MC is under-estimated. To make the analysis less dependent on such effect, a radial cut $R > 12$ cm on the photon impact point at the DCH1 is imposed.

A LKr veto is achieved by rejecting events with additional extra clusters in time with the candidate track ($|\Delta t_{trk-cl}| < 6$ ns) and with energy $E_{veto} > 2$ GeV.

![Gamma energy distribution](image1)

Figure 7: Photon energy distribution for data in the lab. frame.

![Photon radius](image2)

Figure 8: Photon radius at the DCH1: data-MC ratio for $K^+ \rightarrow \pi^0 e^+ \nu$ events.
2.3 Kaon momentum

Charged kaons were delivered with a central momentum of 74 GeV/c and a spread of 1.4 GeV/c (rms). The kaon 3-momentum components were not directly measured for each event but the beam average, measured with reconstructed $K \rightarrow 3\pi$ ($K_{3\pi}$) decays, has been used to compute the $K_{e2\gamma}$ kinematic variables (see [16,17] for further details). The collected data were divided into small time intervals (few hours each) with stable data taking conditions and the distributions of all relevant quantities (central beam momentum, transverse position at the entrance to the vacuum tank and directions) were fitted in each time interval to extract the mean reconstructed values, which were then written in a database. The same procedure has been applied to MC simulated events with one set of constants per run (all events are generated under same conditions within a given run). The beam mean momentum, position and $dy/dz$ slope varied slowly over time, in the ranges of about 0.1 GeV/c, 1 mm and 10 $\mu$rad, respectively (Fig. 9); the mean $dx/dz$ slope, correlated with the magnet spectrometer polarity, varied over time from about $-200\mu$rad to $+200\mu$rad according to the magnet current.

![Figure 9: Mean kaon momentum (left) and kaon trajectory slope $dy/dz$ (right) from database (P5) versus the run number for both data (red points) and MonteCarlo (blue points).](image)

2.4 Corrections applied to the events

Several kinds of corrections are applied to the events in order to improve the resolution on the track momentum as well as on the energy and position of the LKr clusters. These corrections were evaluated for the $R_K$ analysis [6] and studied with dedicated runs and procedures. The main points are briefly summarised below.

2.4.1 LKr energy corrections

- A threshold applied to the calorimeter cells readout introduces a non-linear response between the deposited and measured energy which is not reproduced by the MC sim-

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5If the run conditions changed the current sampling was ended and a new one started.
ulation. A correction is applied to all clusters (only to data) to account for this non-linearity effect. Such correction is negligible for cluster energies above 10 GeV and it can be otherwise parameterized as a polynomial in the measured energy.

- In order to improve the resolution and uniformity of the LKr response and to decrease the electron ID inefficiency, the energy intercalibration of the LKr cells has been studied separately for each period of the data taking by using a sample of electrons from $K_{e3}$ decays. A correction is applied to the data to equalize the LKr response.

### 2.4.2 Corrections to cluster position: projectivity and LKr to DCH alignment

The geometry of the LKr is such that the axis of the calorimeter ionization cells focus to a point $P$ (projectivity structure) at a distance $D = 10998$ cm in front of the LKr plane. The particles originating close to this point impinge on the cells with a trajectory parallel to such axis with the result that the reconstructed transverse coordinates of the clusters do not depend on the longitudinal position of the shower inside the calorimeter. However, if the decay vertex is far from the projectivity point, a small correction to the cluster position depending on the shower depth has to be applied (projectivity correction). In addition, the cluster positions have to be corrected for a LKr to DCH residual misalignment. The complete correction to cluster positions, accounting for both the projectivity and the LKr to DCH misalignment, is different for data and MC events [18,19]. The size of the correction to the transverse cluster positions is usually less than 0.5 cm, while the typical value of the shower depth is about 30 cm.

### 2.4.3 Internal DCH alignment: $\alpha$ and $\beta$ correction

The internal misalignment of the drift chambers and the mis-calibration of the magnetic field in the spectrometer induces a bias in the measurement of the track momentum [20]. A correction is implemented to the measured momentum $P_0$; the effects of misalignment and field mis-calibration are parametrized with the $\alpha$ and $\beta$ parameters, respectively. The corrected track momentum $P_{trk}$ can be obtained using the formula [7,20]:

$$P_{trk} = P_0 \cdot (1 + q \cdot \alpha \cdot P_0) \cdot (1 + \beta)$$

where $q$ is the charge of the track. The absolute values of the $\alpha$ and $\beta$ parameters are of the order of $10^{-5}$ and $10^{-3}$, respectively. In addition, the mean kaon momentum from the database $P_K^0$ has been $\beta$-corrected according to the formula:

$$P_K = P_K^0 \cdot (1 + \beta).$$

The corrections are applied to both data and MC events.

### 2.4.4 Blue Field effect

A stray magnetic field is present in the decay region; it is called “blue” field, after the colour of the tube in which the decay region is contained. It bends the trajectories of charged
particles with respect to their initial direction. The main component of this field is due to the earth magnetic field. The charged tracks are traced backwards through the blue field to the decay vertex position and their directions are corrected according to the measured field.

2.4.5 Correction for kaon momentum spectrum width

The spectra of data and MC kaon momentum differ. The Data/MC ratio from samples of $K_{3\pi}$ events is shown in Fig. 10, in arbitrary units. This shows a difference in the core width (around the nominal value of 70 GeV) and also in the high momentum tail. To correct for this effect, a weight $w$ is applied to MC events:

$$w = 1 + \tilde{\alpha} \cdot (P_{K}^{gen} - 74 \text{GeV/c})^2$$

where $P_{K}^{gen}$ is the generated kaon momentum and $\tilde{\alpha}$ is a time-dependent parameter in the range $-0.08 < \tilde{\alpha} < 0.08$ for P5. Detailed studies and further information can be found in [7,17,21].

Figure 10: Data to MC ratio of the kaon momentum distribution reconstructed using $K_{3\pi}$ events. The formula for the weight is chosen to flatten the parabolic-like shape visible in the data over MC distribution.

2.5 $K_{e2\gamma}(SD^+)$ event reconstruction and definition of the signal region

The reconstruction of the $K_{e2\gamma}(SD^+)$ events is done using the information of the selected track and the cluster associated to the candidate photon. The electron 4-momentum $p_e$ is reconstructed by using the momentum and directions (blue field corrected) of the track measured by the spectrometer in the hypothesis of electron mass, while the photon 4-momentum $p_\gamma$ is computed using the energy and the position of the LKr cluster. The slopes of the
photon trajectory \((dx/dz)_\gamma\) and \((dy/dz)_\gamma\) are reconstructed with the corrected position of the photon cluster \((x_{cl}, y_{cl}, z_{cl})\) and the position of the decay vertex \((x_{vtx}, y_{vtx}, z_{vtx})\):

\[
(dx/dz)_\gamma = \frac{x_{cl} - x_{vtx}}{z_{cl} - z_{vtx}}, \quad (dy/dz)_\gamma = \frac{y_{cl} - y_{vtx}}{z_{cl} - z_{vtx}}.
\]

The gamma 3-momentum \(p_\gamma\) can be expressed as:

\[
p_\gamma = \frac{E_\gamma}{\sqrt{1 + (dx/dz)_\gamma^2 + (dy/dz)_\gamma^2}}((dx/dz)_\gamma, (dy/dz)_\gamma, 1)
\]

The mean kaon 3-momentum \(p_K\) is taken from the database. The \(K_{e2\gamma}\) kinematic variables \(x\) and \(y\) are reconstructed using the formulae (see Sec. 1.1):

\[
x = \frac{2E^*_\gamma}{m_K} = \frac{2p_K \cdot p_\gamma}{m_K^2}, \quad y = \frac{2E^*_e}{m_K} = \frac{2p_K \cdot p_e}{m_K^2}.
\]

In the kaon rest frame the \(SD^+\) events are mainly confined in a kinematic region of the Dalitz plot where the photon and the electron are emitted essentially anti-parallel (Fig. 2-left): they have a relative angle \(\theta^*_e > 120°\) in more than 95% of the cases with the electron energy spectrum peaking at \(E^*(e) = \frac{m_K}{2} \cdot (1 + r_e)\) or \(y = 1 + r_e\) \((r_e \equiv (\frac{m_e}{m_K})^2 = 1.07 \cdot 10^{-6})\). On the other hand, the IB and \(SD^-\) components are concentrated along the line \(x + y = 1\) (Fig. 1). The main backgrounds to \(K_{e2\gamma}\) are due to the \(K_{e3}\) and \(K_{2\pi}\) decays (followed by \(\pi^0 \to \gamma\gamma\) when a photon from \(\pi^0\) remains undetected. The kinematic endpoints of \(K_{e3}\) and \(K_{2\pi}\) backgrounds are (assuming the electron hypothesis for the track): \(y_{max}(K_{2\pi}) = 0.920\) and \(y_{max}(K_{e3}) = 0.925\). A cut \(y > y_{max}(K_{e3})\) is necessary to select the \(SD^+\) term and reject most of the background. The following kinematic cuts define the \(SD^+\) signal region:

- \(|MM^2(e\gamma)| < 0.01 \text{GeV}^2/c^4\), where \(MM^2(e\gamma) = (p_K - p_e - p_\gamma)^2\) is the squared missing mass that for the signal is equal to the neutrino mass (Fig. 11);

- \(x > 0.2\) and \(y > 0.95\). The lower limit in \(x\) is chosen to reduce the IB background that increases at low \(x\), while the lower limit in \(y\) is dictated by the main background coming from \(K_{e3}\) and is chosen in order to minimize such contamination (Fig. 12);
Figure 11: Squared missing mass distribution for data (logarithmic scale). The complete event selection is applied except the $MM^2(e\gamma)$ cut.

Figure 12: Distribution of $x$ and $y$ kinematic variables (Dalitz plot) for $K^+ \rightarrow \pi^0 e^+ \nu$ (left, bkg) and $K^+ \rightarrow e^+ \nu e\gamma$ (right, signal) MC events (logarithmic colour scale).

The number of $K^+ \rightarrow e^+ \nu e\gamma$ candidates selected by the three data analysis at the end of the common event selection is $N_{\text{data}} = 11170$. The study and the evaluation of the background will be described in the following sections.

3 Normalization mode

3.1 $K^+ \rightarrow \pi^0 e^+ \nu$ event selection

The kaon flux is evaluated by using the $K_{e3}$ (followed by $\pi^0 \rightarrow \gamma\gamma$) decay as a normalization channel. This decay has a signature very similar to the signal, with one charged track in the final state and it has been acquired with the same trigger chain\(^6\). It differs from the signal

\(^6\)The $K_{e2}$ main trigger
because of the presence of two photons in the final state, due to the $\pi^0 \rightarrow \gamma\gamma$ decay, instead of a single one. Due to its signature and relatively high branching ratio, the $K_{e3}$ decay is quite easy to select with an almost negligible background contamination ($\sim 10^{-4}$). The charged track selection is the same as for the signal event selection (Section 2.1), so that almost no systematic effects related to the difference between the signal and the normalization channel selections are introduced. The same photon selection reported in section 2.2 is used to select two clusters in the LKr calorimeter which are generated by a $\pi^0 \rightarrow \gamma\gamma$ decay. An additional requirement on the time difference between the two photon clusters being within 3 ns ensures their common origin. The $\pi^0$ reconstruction is eventually achieved by fitting the $\gamma\gamma$ mass distribution (Fig. 13) and keeping events within 5 standard deviations of the distribution core: $125 \text{ MeV} < M(\gamma\gamma) < 145 \text{ MeV}.$

![Figure 13: $\gamma\gamma$ mass distribution for data (logarithmic scale)](image)

The main contribution to the background is coming from the $K^+ \rightarrow \pi^+\pi^0$ decay, with the charged pion mis-identified as a positron. The treatment of the $K_{2\pi}$ background is explained in Sec. 5.1. A kinematic selection of the two decays is achievable by constraining the squared missing transverse momentum calculated with respect to the kaon direction: $p_T > 0.02 \text{ GeV}/c$. In the $K_{e3}$ decay the neutrino in the final state is undetectable and it produces a loss of transverse momentum, while in the $K^+ \rightarrow \pi^+\pi^0$ decay all the particles in the final state are visible by the detector and can be reconstructed. The kinematic separation of $K_{e3}$ and $K^+ \rightarrow \pi^+\pi^0$ is illustrated in Fig. 14 (the distributions are arbitrarily normalised). The $p_T$ spectrum for $K^+ \rightarrow \pi^+\pi^0$ MC events is mainly constrained at zero and is easy to be rejected.

$^7$The contribution to the background coming from the $K^+ \rightarrow \pi^+\pi^0$ decay followed by the $\pi^+ \rightarrow e^+\nu$ decay has been checked to be negligible.
Figure 14: $p_T$ distributions for $K^+ \rightarrow \pi^0 e^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^0$ MC events.

A less significant source of background comes from the $K^+ \rightarrow \pi^0 e^+ \nu$ followed by the $\pi^0 \rightarrow e^+ e^- \gamma$ Dalitz decay: one of the two leptons from the Dalitz decay can be lost, the $\pi^0$ is reconstructed with the photon and the remaining lepton and the $p_{x,0}$ achieved is smaller than the true one. A kinematic rejection of this kind of background is obtained by constraining the reconstructed squared missing mass: $|\left( P_K - P_e - P_{x,0} \right)^2 | < 0.01 \text{GeV}^2/c^4$. The spectra of this quantity are showed in Figs. 15 for the $K^+ \rightarrow \pi^0 e^+ \nu$ MC events (left plot) and for the $K^+ \rightarrow \pi^0 e^+ \nu$ Dalitz MC events (right plot). Both distributions are arbitrarily normalised: the former represents the neutrino squared missing mass and it peaks at 0 GeV$^2/c^4$, the latter shows a large tail at positive values as a consequence of the underestimation of $p_{x,0}$.

3.2 Kaon flux computation

The number of kaon decays (i.e. the kaon flux) can be calculated using the formula:

$$\Phi(K^+) = \frac{N(K_{e3})}{BR(K_{e3}) \times BR(\pi^0 \rightarrow \gamma \gamma) \times Acc(K_{e3}) \times \varepsilon_{K_{e3}}}.$$  (7)
where \( N(K_{e3}) \) is the number of selected \( K^+ \to \pi^0 e^+ \nu \) candidates with the neutral pion decaying in two photons, \( BR(K_{e3}) = (0.0507 \pm 0.0004) \) \cite{9}, \( BR(\pi^0 \to \gamma \gamma) = (0.98823 \pm 0.00034) \) \cite{9}, \( Acc(K_{e3}) \) is the geometrical acceptance and \( \varepsilon_{K_{e3}} \) is the trigger efficiency for \( K_{e3} \) events as described in Sec. 4. The acceptance is computed with MC simulation and is defined as the ratio:

\[
\frac{N_{MC}(K_{e3})}{N_{MC}(-2000 < Z_{gen} < 9000)},
\]

where \( N_{MC}(-2000 < Z_{gen} < 9000) \) is the number of generated \( K^+ \to \pi^0 e^+ \nu (\pi^0 \to \gamma \gamma) \) MC events in a given decay region \((-2000 < Z_{gen} < 9000) \) cm, \( Z_{gen} \) being the \( z \) coordinate of the decay vertex, and \( N_{MC}(K_{e3}) \) is the number of selected \( K^+ \to \pi^0 e^+ \nu (\pi^0 \to \gamma \gamma) \) MC events within the same sample. The \( Z_{vtx} \) cut at the numerator is \(-1600 \) cm \(< Z_{vtx} < 9000 \) cm, namely the same as the one defined in Sec. 2.1 for the signal selection.

Fig. 16 shows the reconstructed quantity \((P_K - P_e)^2\) of \( K^+ \to \pi^0 e^+ \nu (\pi^0 \to \gamma \gamma) \) candidates compared with the sum of normalised estimated signal and main background components: the estimated \( K^+ \to \pi^+ \pi^0 \) events are at a level of \( 10^{-5} \), the \( K^+ \to \pi^0 e^+ \nu \) Dalitz (followed by \( \pi^0 \to e^+ e^- \gamma \)) events are down to \( 10^{-7} \). Several decay channels (\( K_2 \mu_3 \), Dalitz, \( K_{\mu 3} \), \( K_{3\pi} \) Dalitz and \( K_{3\pi} \)) were studied, but no significant background contributions to the signal were found. The residual background to \( K_{e3} \) events was then neglected in the kaon flux measurement.
Referring to the Tab. 3, the values listed in the first two rows are used to compute the kaon flux and its preliminary results are reported in the last row:

<table>
<thead>
<tr>
<th></th>
<th>Angela</th>
<th>Roberto</th>
<th>Stefano</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total $K_{e3}$ data</td>
<td>56,716,791</td>
<td>56,716,899</td>
<td>56,716,520</td>
</tr>
<tr>
<td>Acceptance</td>
<td>(11.265 ± 0.002)%</td>
<td>(11.264 ± 0.002)%</td>
<td>(11.264 ± 0.002)%</td>
</tr>
<tr>
<td>Kaon Flux</td>
<td>(10.10 ± 0.08) $\cdot 10^9$</td>
<td>(10.10 ± 0.08) $\cdot 10^9$</td>
<td>(10.10 ± 0.08) $\cdot 10^9$</td>
</tr>
</tbody>
</table>

Table 3: Numbers of $K^+ \rightarrow \pi^0 e^+ \nu$ candidates, acceptances and kaon flux results.

The flux errors account only for the uncertainties on the quantities appearing in Eq. 7 and are computed propagating the errors in quadrature. The relative error on the flux appears to be $\sim 1\%$, mainly due to the uncertainty on BR($K_{e3}$).

4 Trigger efficiency studies

The trigger used for the acquisition of $K_{e2\gamma}$ events has been originally implemented and optimized during 2007 for the acquisition of $K_{e2}$ events. It consists of coincidences of signals in the two HOD planes (Q1 signal), a loose multiplicity cut on the number of hits in the drift chambers (1TRK-LM signal) and an energy deposition in the LKr electromagnetic calorimeter of at least 10 GeV ($E_{LKr}(10$ GeV signal).

The trigger efficiency has been directly measured with a control data sample by means of Pattern Units (PU): a digital register which allows to retrieve\(^8\) the trigger conditions during the data acquisition. The bits corresponding to the relevant trigger sub-signals are:

- bit 1 in PU channel 6 for the $E_{LKr}(10$ GeV) trigger;
- bit 9 in PU channel 4 for the 1-TRKLM trigger;
- bit 7 in PU channel 4 for the Q1 trigger.

The so called $K_{e2}$ trigger word (TW) bit, referring to the overall $K_{e2\gamma}$ trigger condition corresponds to 0x0400 (hex. code). The control data sample is selected with a control trigger signal provided by the neutral hodoscope (NHOD) which triggers on a shower in the LKr. The NHOD trigger rate is higher compared with the triggers to be measured and the control data sample has been downscaled by a factor of 150 with no significant consequences on the precision of the efficiency measurements. The trigger efficiency for the sub-signal $i$ (e.g Q1) is defined as:

$$\varepsilon_i = \frac{N_{PU(i)*NHOD}}{N_{NHOD}} \quad (9)$$

where $N_{NHOD}$ is the number of selected events in the control data sample and $N_{PU(i)*NHOD}$ is the number of events reconstructed and selected by the trigger sub-signal $i$ in coincidence

\(^8\)Each bit of the registered word corresponds to a trigger condition and is set to one when this is fulfilled.
with the control trigger signal NHOD. The total trigger efficiency is measured by requiring
the coincidence of the three sub-signals taken from the PU and the control trigger signal
NHOD:

\[ \varepsilon = \frac{N_{PU(Q1)\times PU(TRK)\times PU(LKr)\times NHOD}}{N_{NHOD}} \]  

The measurement of the \(K_{e2\gamma}\) trigger efficiency \(\varepsilon_{K_{e2\gamma}}\) is constrained by the statistically poor
data sample of \(K_{e2\gamma}\) candidates (\(\sim 10000\) events). As a first approach the trigger efficiency
has been estimated for the normalization channel (\(K_{e3}\)) only. The data sample of \(K_{e3}\)
candidates comprises about 300000 events and the measured efficiencies for the single trigger
sub-signal components and the main trigger are reported in the following sections.

4.1 Q1 trigger efficiency

The Q1 signal is part of the main \(K_{e2}\) trigger; it requires at least 1 time coincidence between
a signal from a vertical scintillator and a signal from a horizontal scintillator in the same
hodoscope quadrant. A source of Q1 inefficiency is geometrical: small "cracks" exist in the
positions corresponding to the end of the horizontal and vertical scintillator bars. Fig. 17
shows the Q1 trigger inefficiency for \(K_{e3}\) events plotted as a function of the track impact
point on the hodoscope XY front plane: the "cracks" generated by the gaps between the
scintillators are visible. The Q1 trigger efficiency estimated by using \(K_{e3}\) events is:

\[ \varepsilon_{Q1} = 0.99725 \pm 0.00009_{\text{stat}} \]

Figure 17: The Q1 inefficiency as a function of the track impact point on the HOD, evaluated
using \(K_{e3}\) events.
4.2 $E_{LKr}(10 GeV)$ trigger efficiency

The $E_{LKr}(10 GeV)$ trigger signal requires a minimum energy deposit in the LKr calorimeter. The selection criteria addressed in chapter 2.1 for both $K_{e2\gamma}$ and $K_{e3}$ channels have been chosen in order to guarantee an energy deposition in the LKr calorimeter far above the 10 GeV trigger energy threshold. The $E_{LKr}(10 GeV)$ trigger efficiency for $K_{e3}$ events in bins of the sum between the track and the $\pi^0$ energy depositions at the LKr calorimeter is shown in Fig. 18: the efficiency is very high and flat on the full energy range. The $E_{LKr}(10 GeV)$ trigger efficiency estimated by using $K_{e3}$ events is:

$$\varepsilon_{E_{LKr}(10 GeV)} = 0.99991 \pm 0.00002_{stat}$$

Figure 18: The $E_{LKr}(10 GeV)$ trigger efficiency for $K_{e3}$ events as a function of the sum between the track and the $\pi^0$ energy depositions in the LKr calorimeter.

4.3 1TRK-LM trigger efficiency

The 1TRK-LM signal is part of the $K_{e2}$ main trigger and allows the suppression of events with high activity on the drift chambers (cuts on number of hits on DCH 1,2 and 4). The 1TRK-LM trigger condition is highly efficient for $K_{e3}$ events: Fig. 19 shows that it is independent on the energy of the photon produced in the $\pi^0 \rightarrow \gamma\gamma$ decay (left plot) and its distribution as a function of the track impact point on DCH1 (right plot). As further checks, the 1TRK-LM trigger efficiency for the $K_{e3}$ selection has been also studied as a function of several variables of the analysis and no dependences have been observed (Fig. 20). The 1TRK-LM trigger efficiency estimated by using $K_{e3}$ selected events is:

$$\varepsilon_{1TRK-LM} = 0.99869 \pm 0.00006_{stat}$$
Figure 19: The 1TRK-LM trigger efficiency for $K_{e3}$ events as a function of the photon energy (from the $\pi^0$ decay) (left) and the inefficiency as a function of the track impact point on DCH4 (right).

Figure 20: The 1TRK-LM trigger efficiency for $K_{e3}$ events vs the track radius (left) and the photon radius (right) at the DCH1 front plane.

4.4 Total trigger efficiency

The efficiency of the main trigger for $K_{e3}$ events ($\varepsilon_{K_{e3}}$) is reported in Tab. 4; the results achieved with the three ongoing analyses are presented. $\varepsilon_{K_{e3}}$ does not depend on the relevant variables of the analysis for the normalization channel. Fig. 21 shows that $\varepsilon_{K_{e3}}$ does not exhibit any dependence on the reconstructed track momentum and kaon decay vertex: this is confirmed by the constant fits applied in the plots. It has been also studied as a function of relevant variables in common with the $K_{e2\gamma}$ analysis, such as the photon energy and the
$y$ kinematic variable: no dependence is observed (Fig. 22). A check on the stability of the trigger efficiency during the data taking (P5) has been performed and showed no further dependence on the run number (Fig. 23).

<table>
<thead>
<tr>
<th>Trigger Efficiency</th>
<th>Angela</th>
<th>Roberto</th>
<th>Stefano</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.99481 ± 0.00013</td>
<td>0.99457 ± 0.00012</td>
<td>0.99473 ± 0.00013</td>
</tr>
</tbody>
</table>

Table 4: Total trigger efficiency $\varepsilon_{K_{e3}}$ results. The total efficiency is measured by requiring the coincidence of the three sub-signals taken from the PU and the control trigger signal NHOD (Eq. 10).

Figure 21: $K_{e3}$ trigger efficiency plotted vs the reconstructed track momentum (left) and the reconstructed longitudinal decay vertex (right): no dependence is observed for P5 sample.

Figure 22: $K_{e3}$ trigger efficiency vs the photon energy (left) and the $y$ variable (right) for P5 sample.
4.5 Preliminary conclusions

The evaluation of the $K_{e2\gamma}$ trigger efficiency is affected by the large downscaling $D=150$ of the control trigger NHOD which limits the statistics available for trigger studies (about 100 $K_{e2\gamma}$ events). To overcome this issue, at this stage of the analysis, the trigger efficiency has been evaluated using the $K_{e3}$ normalisation channel, which has a similar event topology of the $K_{e2\gamma}$ signal. As the trigger inefficiency is clearly dominated by the Q1 trigger component (coincidences of signals in the two HOD planes), it is reasonable to assume the trigger efficiency for the signal events at the same level of the measured one for the normalisation channel. The $K_{e3}$ main trigger efficiency and its sub-signal efficiencies have been studied as a function of several relevant variables of the analysis: no dependence with respect to all the variables is observed.

4.6 Trigger efficiency for backgrounds

The trigger efficiency has been also studied using the $K_{e2\gamma}$ selection and a source of inefficiency coming from the 1TRK-LM sub-signal has been found. The measurement of the 1TRK-LM trigger efficiency for $K_{e2\gamma}$ events shows that the sub-signal is affected by background events to $K_{e2\gamma}$ with lost photons, such as $K_{e3}$ and $K_{2\pi}$. This is due to lost photons interacting with the material of the detector, showering and thus producing high multiplicity in the drift chambers; such events are most likely rejected by the 1TRK-LM trigger component. Moreover, due to the correlation between the energy and the direction of particles, a dependence of this inefficiency on the energy of the detected photon is observed; namely the inefficiency increases when the detected photon has low energy (the lost photon has most of the $\pi^0$ energy and is nearly parallel to the beam pipe).

The 1TRK-LM trigger efficiency has been measured using the $K_{e2\gamma}$ event selection (Sec. 2) with no kinematic cuts on the missing mass and $x,y$ variables applied; the sample is dominated by the background. Fig. 24 shows the 1TRK-LM trigger efficiency as a function of the
photon energy (left) and on the Dalitz plot (right). The selected events are mainly $K_{e3}$ and $K_{2\pi}$ backgrounds with a lost photon; the dependence of the inefficiency on the photon energy is visible and the inefficiency increases up to 10% at low energies for the detected gamma. To properly evaluate the residual background to $K_{e2\gamma}$, this dependence, not simulated by the MC, must be taken into account by weighting the MC spectra for the 1TRK-LM trigger efficiency on a bin-by-bin basis. Further studies on the 1TRK-LM inefficiency are needed to estimate the precision at which it is controlled; the major limitation for this kind of studies is the low statistics in the $K_{e2\gamma}$ signal region that prevents a detailed study of the trigger inefficiency affecting the backgrounds.

![Graph showing 1TRK-LM trigger efficiency vs gamma energy and on the Dalitz plot](image)

Figure 24: The 1TRK-LM trigger efficiency vs gamma energy (left) and on the Dalitz plot (right) computed using the $K_{e2\gamma}$ event selection with no kinematic cuts (missing mass, $x$ and $y$). The contribution from background events to $K_{e2\gamma}$ (mainly $K_{e3}$ and $K_{2\pi}$ decays) is visible and the inefficiency due to lost photons increases up to 10% at low energies.

The trigger inefficiency affecting background events to $K_{e2\gamma}$ obviously induces a large effect on the measurement of the $K_{e2\gamma}$ form factors for which the shape of the distribution is important. More detailed studies are foreseen to estimate the systematics related to this effect and to assess to what precision it is known; these issues will be addressed in a subsequent note.

5 $K^+ \rightarrow e^+ \nu_e \gamma$ Data-MC Comparison and Background Evaluation

The MC samples used in this analysis have been generated using the CMC framework, which allows the user to choose the theoretical model for the generation of the decay. The $K_{e2\gamma}$ (SD+) MC has been simulated according to the form factors measured by KLOE (Eq. 6).
The main source of background to the $K_{e2\gamma}$ decay is coming from the $K_{e3}$ channel. Its branching ratio $BR(K_{e3}) = (5.07 \pm 0.04)\%$ is a factor of $\sim 3000$ higher than that of the signal ($BR(K_{e2\gamma}, SD^+) \sim 10^{-5}$) and its final state is quite similar to that of the signal. A $K^+ \to \pi^0 e^+\nu$ decay may lead to a $K^+ \to e^+\nu_e\gamma$ signature if one photon from the $\pi^0$ decay is undetected (absorbed before the LKr, or out of the calorimeter acceptance) or if the $\pi^0$ decays in the Dalitz mode $\pi^0 \to e^+e^-\gamma$ and the $e^+e^-$ pair is undetected.

Another significant source of background to the $K_{e2\gamma}$ decay is coming from the $K_{2\pi}$ channel. Its branching ratio $BR(K_{2\pi}) = (20.66 \pm 0.08)\%$ is a factor of $\sim 12000$ higher than that of the signal and their final states are quite similar. A $K^+ \to \pi^+\pi^0$ decay contributes to the background if one photon from the $\pi^0$ decay is undetected and one of these two mechanisms happens:

- the charged pion is mis-identified as a positron because it completely releases its energy in the LKr calorimeter; this contribution has been addressed with the approach explained in Sec. 5.1.
- the charged pion decays to a positron through the process $\pi^+ \to e^+\nu_e$ ($BR \sim 10^{-4}$). This contribution has been specifically studied with a $K_{2\pi}$ MC sample in which the charged pion decays to a positron and is found not to be less than ??? of the signal.

Another source of background to $K_{e2\gamma}$ events is coming from the $K^+ \to \pi^+\pi^0$ followed by the $\pi^0$ decay in the Dalitz mode $\pi^0 \to e^+e^-\gamma$. Two different components of this background can be distinguished according to the way this process may lead to a $K_{e2\gamma}$ signature:

- the $\pi^+$ is mis-identified as a positron and the $e^+e^-$ pair is undetected;
- the charged pion and the electron are both undetected.

The latter contribution has been addressed with a specific $K_{2\pi}$ (followed by $\pi^0 \to e^+e^-\gamma$) MC sample and is found not to be relevant. The former contribution has been studied with the same MC sample using the approach explained in Sec. 5.1.

The $K^+ \to e^+\nu_e\gamma$ (IB) component has obviously the same final state of the signal but the radiative photon is soft, its energy spectrum is different from the one emitted in the SD modes and this background is reduced to a negligible amount using kinematic and geometrical cuts.

Background channels with muons in the final state, such as the $K_{\mu3}$ and its Dalitz mode ($K_{\mu3}$ followed by $\pi^0 \to e^+e^-\gamma$), have also been considered. These channels may contribute to the background to $K_{e2\gamma}$ events if the charged muon is mis-identified as a positron and in one case a photon from the $\pi^0$ decay is undetected (absorbed before the LKr, or out of the calorimeter acceptance), while in the other case the $\pi^0$ decays in the Dalitz mode $\pi^0 \to e^+e^-\gamma$ and the $e^+e^-$ pair is undetected. A $\mu^+$ can be mis-identified as a positron if:

- it completely releases its energy in the LKr calorimeter through a "catastrophic" bremsstrahlung; this contribution being addressed with a similar approach to the one used for pions and explained in Sec. 5.1.
- it decays to a positron via the process $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$. 

27
Both background contributions from $K_{e3}$ and its Dalitz mode to $K_{e2\gamma}$ decay have been studied with specific MC samples, with and without the muon decaying to a positron; both were found to be negligible.

The number of $K_{e2\gamma}(SD^+)$ candidates, its acceptance\(^9\) and the expected number of residual events for the main background components are shown in Tab. 5.

<table>
<thead>
<tr>
<th>Total $K_{e2\gamma}(SD^+)$ data</th>
<th>Angela</th>
<th>Roberto</th>
<th>Stefano</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{e2\gamma}$ Acceptance</td>
<td>0.06878 ± 0.00004</td>
<td>0.06878 ± 0.00004</td>
<td>0.06876 ± 0.00004</td>
</tr>
<tr>
<td>$N(K_{e3})$ bkg</td>
<td>270 ± 24</td>
<td>273 ± 24</td>
<td>275 ± 24</td>
</tr>
<tr>
<td>$N(K_{2\pi})$ bkg</td>
<td>163 ± 4</td>
<td>173 ± 4</td>
<td>165 ± 4</td>
</tr>
<tr>
<td>$N(K_{e3}$ Dalitz) bkg</td>
<td>3.3 ± 0.9</td>
<td>3.4 ± 0.9</td>
<td>3.3 ± 0.9</td>
</tr>
<tr>
<td>$N(K_{2\pi}$ Dalitz) bkg</td>
<td>1.2 ± 0.1</td>
<td>1.4 ± 0.1</td>
<td>1.2 ± 0.1</td>
</tr>
</tbody>
</table>

Table 5: Summary of the most relevant backgrounds to $K_{e2\gamma}(SD^+)$. 

The residual backgrounds are evaluated using the formula\(^10\):

$$N_{bkg} = \Phi(K^+) \times \sum_{bkg_i} BR(bkg_i) \times Acc(bkg_i)$$  \hspace{1cm} (11)

where $\Phi(K^+)$ is the kaon flux, $BR(bkg_i)$ is the branching ratio of the single decay channel contributing to the background and $Acc(bkg_i)$ is the corresponding geometrical acceptance. 

All acceptances for background and signal processes are computed, as for the normalization channel (Eq. 8), by MC simulations and defined with the ratio between selected MC events and generated MC events in the decay region between \((-2000 < Z_{gen} < 9000)\) cm. The errors associated to the residual backgrounds, reported in Tab. 5, are the total errors computed by error propagation of the terms in Eq. 11.

The reliability of the MC simulation has been checked using the $K_{e2\gamma}$ event selection. All the MC distributions have been normalised by means of the kaon flux and BRs. The kaon flux is computed using the $K_{e3}$ normalization channel; the BR assumed for the $K_{e2\gamma}$ $(SD^+)$ MC sample normalisation is:

$$BR(K^+ \rightarrow e^+\nu_{e}\gamma SD^+) = (1.52 \pm 0.23) \times 10^{-5}$$  \hspace{1cm} (12)

corresponding to the full kinematic region [8]. Figs. 25, 26, 27 and 28 show some MC/Data comparisons for several relevant variables of the $K_{e2\gamma}$ analysis. The agreement between data and MC will be studied using control samples (side-bands of applied cuts) which are background enriched. This will be reported in a final subsequent note, also including several

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\(^{9}\)The signal acceptance is evaluated by MC assuming the form factors as measured by KLOE (Eq. 6).

\(^{10}\)As a direct consequence of the study presented in Sec. 4, the trigger efficiency is assumed to be the same as the one used for the kaon flux computation in Sec. 3.2. It thus cancels out and is not explicitly reported in the formula.
stability checks of the measurement with respect to the variation of all relevant cuts. The procedure will help quantifying the MC/Data agreement, thus validating the MC simulation at a certain level of accuracy.

Figure 25: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Track momentum (left) and photon energy (right) in the lab. frame.

Figure 26: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Longitudinal kaon decay vertex (left) and track radius at DCH1 (right)
Figure 27: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Kinematic variables $x$ (left) and $y$ (right). The Data/MC ratio for the $x$ variable is incompatible with a constant; it is strongly dependent on the values of the form factors used in the signal simulation (in the present analysis the KLOE values have been used).

Figure 28: Reconstructed squared missing mass $MM^2(e\gamma)$ distributions of $K^+ \rightarrow e^+\nu_e\gamma$ (SD$^+$) candidates compared with the sum of normalised estimated signal and background components. Linear scale (left) and logarithmic scale in a wider $MM^2(e\gamma)$ range (right). The visible shift between data and MC (left) is under investigation. The deficit of reconstructed MC events in the region below 0.01 GeV$^2$/c$^4$ (right) is mostly outside of the signal region and is, at least in part, due to resolution effects dominated by non gaussian tails. A detailed study of such effects and a quantitative estimation of systematics accounting for them is foreseen in the next note.
From the table 5 it can be inferred that the main background component is due to $K_{e3}$ and its background to signal ratio is estimated to be $B/S(K_{e3}) \sim 2.5\%$. This amount goes up to $B/S(K_{e3}) \sim 6.5\%$ when loosening the $y$ cut from 0.95 to 0.94 because the resolution of the detector smears the $K_{e2\gamma}$ endpoint from the kinematic limit $y_{max}^{K_{e3}} \sim 0.93$ up to $y \sim 0.96$. Although a tight cut on $y$ decreases the amount of $K_{e3}$ residual background, it follows a reduction of the signal region available for the measurement of the branching ratio and for the extraction of the form factors. A quantitative study on the $y$ cut will be done in the near-future developments of the analysis in order to find the best compromise between the maximum $B/S$ tolerable and a significant signal acceptance. A more detailed study of both backgrounds due to $K_{e3}$ and $K_{2\pi}$ events is also ongoing to optimise the background subtraction, which might be dominating the future total uncertainty.

5.1 Particle mis-identification probability

As the interaction and clusterisation of hadrons inside the LKr calorimeter are poorly described by CMC, the background contributions to $K_{e2\gamma}$ events coming from the $K_{2\pi}$ decay and its Dalitz mode, with the $\pi^+$ mis-identified as a positron, have been addressed with the following approach. Specific $K_{2\pi}$ and $K_{2\pi}$ (followed by $\pi^0 \rightarrow e^+e^-\gamma$) MC samples, in which the pion does not decay, have been simulated and analysed with the signal event selection. The simulation of the energy released by the charged pion in the LKr calorimeter is inadequate for a particle ID based on the reconstructed ratio $E/p$, as described in section 2.1. The cut $0.95 < E/p < 1.1$ on the LKr cluster associated to the track is not applied for these background MC samples with a charged pion in the final state and MC events are then weighted with appropriate measured values for the pion mis-ID probability as electron $P(\pi \rightarrow e)$, which is momentum dependent and has a typical value of $\sim 5 \times 10^{-3}$ [22].

As the simulation of the interaction of muons with the LKr calorimeter suffers of the same problem discussed for the pions, the background contributions to $K_{e2\gamma}$ events coming from the $K_{\mu3}$ decay and its Dalitz mode, with the $\mu^+$ mis-identified as a positron, have been addressed with a similar approach. Specific $K_{\mu3}$ and $K_{\mu3}$ (followed by $\pi^0 \rightarrow e^+e^-\gamma$) MC samples, in which the muon does not decay, have been simulated and analysed with the signal event selection. The cut $0.95 < E/p < 1.1$ on the LKr cluster associated to the track is not applied for these background MC samples with a muon in the final state and MC events have been weighted with the measured value for the muon mis-ID probability as electron $P(\mu \rightarrow e)$, which is momentum dependent and has a typical value of $\sim 10^{-6}$ [23].

6 $K_{e2\gamma}(SD^+)$ form factors measurement

As described in Sec. 1.1, the $K_{e2\gamma}(SD^+)$ decay amplitude can be parameterized by the vector and axial-vector form factors, $F_V(x)$ and $F_A(x)$ respectively. The evaluation of the $K_{e2\gamma}(SD^+)$ form factors requires a theoretical effort due to the non-perturbative behaviour of the strong interactions at low energies. In this respect, ChPT provides a universal framework for treating in a perturbative way, leptonic, semi-leptonic and non-leptonic decays, including
rare and radiative modes; in particular, it gives a clear and unambiguous prediction of the $K_{e2\gamma}$ form factors. Recent estimations [5] assign theoretical uncertainties of 6% on $F_V(0)$ and 1% on $F_A(0)$, while the linear term $\lambda$ in ChPT $O(p^6)$ is calculated with a 33% accuracy. A measurement of the $K_{e2\gamma}$ form factors with a precision below the theoretical one represents an important test of ChPT. The description of the fitting method and the preliminary results of the form factor measurement will be discussed below.

6.1 Definition of the measurement and fitting procedure

A procedure has been established to measure the form factors. The number of $K_{e2\gamma}$ ($SD^+$) decays predicted by theory in a given $i$ bin of $x$ ($N_{i\text{theo}}^i$) is computed according to the theoretical differential decay rate with the form factors as given by ChPT at $O(p^6)$. The number of events is normalized using the kaon flux $\Phi(K^+)$:

$$N_{i\text{theo}}^i = \Phi(K^+) \cdot dBR_i$$

where $dBR_i = \frac{dBR}{dx} \cdot \Delta x$ is the $SD^+$ theoretical differential branching ratio evaluated at bin centre and multiplied by the bin width $\Delta x$; at $O(p^6)$ it depends on $F_V(0)$, $F_A(0)$ and $\lambda$. The reconstructed distribution of the signal is obtained by convoluting (folding) the theoretical spectrum with the detector resolution and acceptance as simulated by the MC. The effects of resolution and acceptance are encoded in a folding matrix $P_{i'j'}$ (left plot in Fig. 29) that is defined as the number of the $SD^+$ MC events $N_{i'\text{rec}}^{i'}$ generated in bin $i'$ and reconstructed in bin $j'$ over the number of the total events generated in bin $i'$:

$$P_{i'j'} = \frac{N_{i'\text{rec}}^{i'}}{N_{i'\text{gen}}}$$

The matrix $P_{i'j'}$ is thus the conditional probability that an event will be found with measured value $x$ in bin $i'$ given that the true value was in bin $j'$ (Fig. 29). The primed indices reflect the fact that, in principle, the binning used for the convolution can be different with respect to the one used for the fit. $P_{i'j'}$ depends on the theoretical model implemented in the MC, which is a priori unknown; this dependence cancels out if the binning used in the computation of $P_{i'j'}$ is small enough that the resolution is approximately constant over the bin $i'$. The expected number of $K_{e2\gamma}$ ($SD^+$) events is showed in Fig. 29 (right plot) and is computed as:

$$N_{i\text{exp}}^i = \Phi(K^+) \cdot \sum_{i' \in \text{bin } i} \left( \sum_{k'} dBR_{k'} \cdot P_{k'j'} \right) ,$$

where the sum in the brackets extends over all the convolution bins, while the other runs only over the convolution bins inside the fit bin $i$. The total number of background events in bin $i$ ($N_{i\text{bkg}}^i$) is estimated according to the Eq. 11 in each bin $i$, and is subtracted to the number of data $N_{i\text{data}}^i$, the latter is then compared with the expected number of $K_{e2\gamma}$ ($SD^+$) events.

\footnote{The signal MC sample is generated assuming the form factors as measured by KLOE [10].}
The best value of the form factors compatible with the data is found by minimizing the $\chi^2$ function:

$$\chi^2 = \sum_{i=1}^{N} \frac{(N^i_{\text{data}} - N^i_{\text{exp}})^2}{\sigma^2_{N^i_{\text{data}}} + \sigma^2_{N^i_{\text{exp}}}}$$

assuming the $\sigma^2_{N^i_{\text{exp}}}$ to be negligible. The choice of the bin width used for the fit is dictated by the limited statistics of the $SD^+$ candidates. The $x$ distribution has been divided in bins of width $\Delta x = 0.1$ (about 10 times larger than the average $x$ resolution) and the fit has been performed over 8 bins ($0.2 < x < 1$). The average number of candidates falling in each bin is about 1000 events/bin (Fig. 31).

Figure 29: (Left) Folding matrix of the $x$ distribution. Events reconstructed with an energy lower than the true energy are visible. (mis-reconstructed events) (Right) Distribution of the $K_{\epsilon2\gamma}$ candidates over $x$.

### 6.2 Preliminary results

#### 6.2.1 Fit to the LFQM

In Fig. 30 data are overlaid with the LFQM prediction which features a complicated dependence of $F_V$ and $F_A$ on $x$, with $F_V = F_A = 0$ at $x = 0$. An index of the agreement is given by the $\chi^2$ obtained by fitting the LFQM spectrum to data without free parameters. The data are clearly in disagreement with this model as the overlay gives $\chi^2/ndf = 2850/8$. At this stage of the analysis, LQFM is excluded by data. However, there is room for improvements allowing free parameters, as $F_V(0)$ and $F_A(0)$, in the fitting procedure.
Figure 30: Data are overlaid to LFQM prediction. Only the main backgrounds $K_{e3}$ and $K_2\pi$ are shown. LFQM is excluded by data as the overlay gives $\chi^2/ndf = 2850/8$.

6.2.2 Fit to the ChPT at $O(p^6)$

The result of the fit using ChPT at $O(p^6)$ with $F_V(0)$ and $\lambda$ as free parameters (see Sec. 1) is shown in fig. 31. The fit gives $\chi^2/ndf = 5.5/6$ with a correlation between parameters $\rho = -0.94$.

Figure 31: Fit to ChPT at $O(p^6)$. Only the main backgrounds $K_{e3}$ and $K_2\pi$ are shown. The fit parameters are $F_V(0)$ and $\lambda$ (see Sec. 1) while the axial form factor is fixed to its $O(p^6)$ value $F_A(q^2) = F_A(0)$.  

7 Conclusions

A comprehensive study of the process $K^+ \to e^+\nu_e\gamma$ (SD+) is being performed with the partial (P5) data sample collected in 2007. A signal acceptance of $\sim 7\%$ is achieved and the sample comprises about 10k candidates with a background contamination of $\sim 5\%$ mainly due to the $K_{e3}$ and $K_{2\pi}$ decay channels. A $\chi^2$ fit has been performed to the measured $x$ spectrum using the distribution expected from the ChPT models. The estimated form factor parameters with their uncertainties and the correlation coefficients are not presented in this note. They are expected to receive a significant contribution from a systematic uncertainty, mainly due to the background subtraction, while the statistical uncertainty $(\Delta F_V(0))_{\text{stat}} \sim 0.002$ and $(\Delta \lambda)_{\text{stat}} \sim 0.06$ benefits from the large number of candidates, a factor of $\sim 10$ higher than the one analysed by the KLOE collaboration. NA62 results aim to a model independent extraction of the theoretical form factors, which will allow validating or falsifying the LFQM, as well as determining the ChPT parameters involved with unprecedented accuracy.

References


