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$\pi\mu$ atoms from $K_{\mu 4}$ decay in NA62

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Abstract

We propose to measure the $\pi\mu$ atoms ($A_{\pi\mu}$) yield in experiment NA62. The rate of $\pi\mu$ atoms formation in $K_{\mu 4}$ decay is compatible with the $K^+ \to \pi^+\nu\bar{\nu}$ decay rate, the registration of both decays in STRAW detector are similar, and their missing mass regions are neighboring. So the detection of $A_{\pi\mu}$ atoms by NA62 could be a physical reference point to demonstrate the NA62 ability to measure a single-track mode with the decay rate of the order of 10^{-10}.

1 Introduction

In the beginning of seventies L.Nemenov [1] proposed to measure the $\pi\mu$ atoms formation in the decay $K_L^0 \to \pi\mu\nu$ i.e. the decay $K_L^0 \to A_{\pi\mu}\nu$. In 1976 such experiment has been done at BNL [2] detecting 18 events of this rare decay. It was the first observation of an elementary atom composed from two unstable particles. Ten years later at Fermilab [3] the 320 $\pi\mu$ atoms were detected and the first measurement of $A_{\pi\mu}$ formation probability in the $K_L^0 \to \pi\mu\nu$ decay has been published:

$$R = \frac{\Gamma(K_L^0 \to A_{\pi\mu}\nu)}{\Gamma(K_L^0 \to \pi\mu\nu)} = (3.9 \pm 0.39) \times 10^{-7}$$

.

As was shown in [4], the branching ratio for $A_{\pi\mu}$ atom formation via $K_{\mu 4}$ decay $Br(K^+ \to \pi^+ A_{\pi\mu}\nu) \approx 0.5 \times 10^{-10}$ is compatible with the Standard

Model prediction value for our goal mode $Br(K^+ \to \pi^+\nu\bar{\nu}) \approx (0.85 \pm 0.07) \times 10^{-10}$.

$A_{\pi\mu}$ atom is a neutral system with a lifetime determined by the charged pion lifetime $\tau_{\pi^+} = 2.6 \times 10^{-8}s$. Positive pions from the atom formation events would be a problematic background for $K^+ \to \pi^+\nu\bar{\nu}$ decay, if $A_{\pi\mu}$ would not cause an additional signals in the detector, and if the corresponding missing mass calculated from the positive pion track could be significantly less than the atom production threshold $(m_\pi + m_\mu)$. As was shown in [4], only $\sim 15\%$ of $A_{\pi\mu}$ atoms are produced in the kinematical region $(m_\pi + m_\mu)^2 \leq (p_K - p_\pi)^2 \leq 4m_\pi^2$, where they can imitate $K^+ \to \pi^+\nu\bar{\nu}$ decay in STRAW detector.

$A_{\pi\mu}$ detection in NA62 would give a possibility to check the experiment ability to measure the properties of the mode that is well predictable theoretically, has the rate of the same order as $K^+ \to \pi^+\nu\bar{\nu}$ decay one and assumes the experimental difficulties of the similar nature.

The differential rate of the decay $A_{\pi\mu}$ in kaon decay is described by the expression [4] :

$$\frac{d\Gamma(K^+ \to \pi^+ A_{\pi\mu}\nu)}{dE_\pi dE_\nu} = \frac{G_F^2 V_{us}^2}{m_\pi(4\pi m_K)^3}\frac{1.2\alpha^3\mu^3}{\pi}\Phi(E_\pi, E_\nu), \quad (1)$$

$$\begin{aligned}
\Phi(E_\pi, E_\nu) &= q_1(q_2 + 2m_\pi m_a) \mid F \mid^2 + q_1(q_2 - 2m_\pi m_a) \mid G \mid^2 + m_\mu^2 q_3 \mid R \mid^2 \\
&+ 2(q_1 q_2 - 2m_\pi^2 q_3)Re(FG^*) + 2m_\mu(m_a q_1 + m_\pi q_3)Re(FR^*) \\
&+ 2m_\mu(m_a q_1 - m_\pi q_3)Re(RG^*) + \frac{m_\pi^2}{m_a^2}\left(4E_\nu E_\pi q_1 - q_1^2 - 4m_\pi^2 E_\nu^2\right) \\
&\times \left(\frac{q_3}{m_\pi^2} \mid H \mid^2 - 2\frac{m_a}{m_\pi}Re(GH^* + FH^*)\right), \quad (2)
\end{aligned}$$

where

$$\begin{aligned}
q_1 &= 2p_1 k_2 = m_K^2 + m_a^2 - m_\pi^2 - 2m_K E_a \\
q_2 &= 2p_1 p_a = m_K^2 - m_a^2 - m_\pi^2 - 2m_K E_\nu \\
q_3 &= 2p_a k_2 = m_K^2 - m_a^2 + m_\pi^2 - 2m_K E_\pi \quad (3)
\end{aligned}$$

A simple parameterizations from [5] are used here for the vector H and axial F,G,R form factors.

2 Toy Monte Carlo simulation

To estimate the expected size of the $\pi\mu$ atom signal in the NA62 setup we use the simplified fast MC program, written initially for the preliminary estimations of the straw detector geometrical efficiencies, rates and trigger capabilities. For the data analysis stage, the simulation have to be done with the complete Monte Carlo code, that is currently under development in Collaboration [6].

Longitudinal positions of the straw chambers and spectrometer magnet are taken from the current BEATCH file [7]. Transversal distributions of the beam particles at the straw chambers are approximated by three-Gaussian function and are tuned to the results of TURTLE-based beam simulation [8, 9] (Figs. 1 and 2). Beam charged particles momenta after the final collimator are simulated using the 5-Gaussian parametrization of the TURTLE-based beam simulation results [8, 9] (Fig. 3). A common additional initial transversal momentum of 0.09 GeV is added to all the beam particles momenta in order to steer the beam according to the setup layout.

Momenta of the particle both in ZX and ZY projections are slightly deviated from the beam central direction to simulate a small divergence, needed to reproduce the width change of the transversal distributions between the chambers (Figs. 1 and 2). No other momentum-coordinate correlations were reproduced. Flight of the beam kaon or pion is simulated according to the corresponding lifetime and mass. Than a decay is sampled (only $\pi^{\pm} \rightarrow \mu^{\pm}\nu$ for charged pions), if it happens before the last straw chamber for accidental component, or if it was inside the fiducial region for the signal decay.

Only charged decay products are taken into account, and only their direct flight and transversal momentum kick of -0.270 GeV at the spectrometer magnet center plane are simulated. Scattering, showers, delta-electrons *etc* were not reproduced.

Straw chambers geometry excludes the straw presence in every view inside the circle with a 6 cm radius, that is centered at the path of the "central" beam particle with a momentum of 75 GeV and initial zero transversal coordinates (Fig. 4). For the 4-th chamber this excluding circle is shifted additionally by -1 cm (to improve the asymmetric Ke4 decays rejection).

The hit presence is detected as a crossing in space between the track path and the straw. 3% straw overall inefficiency is applied. Additional efficiency drop near the straw edge is taken into account according to the inefficiency simulation made on the basis of GARFIELD model [10]. Apart from that, the extra diminishing of the efficient straw radius by 200 μm have been simulated to take into account the worst limit of the detector efficiency losses due to the straw curvatures.

In order to check the hits simulation, a sample of randomly triggered events has been produced in the time window of 125 ns width (that is enough for the electrons drift in the fast gas mixture Ar 70% CO2 30%) for the 570 MHz beam rate (with a 8% of kaons).

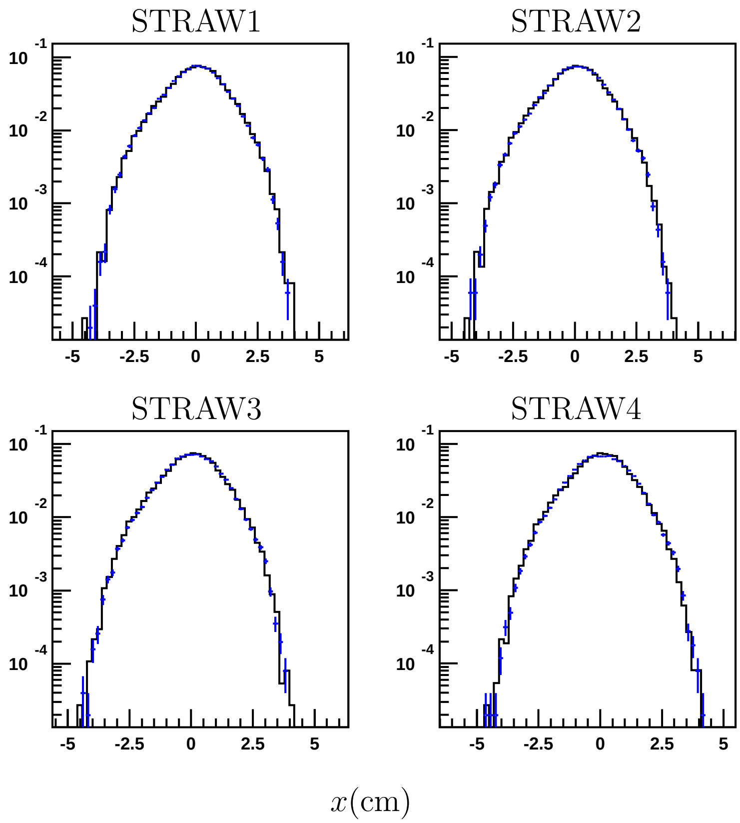

Figure 1: Normalized beam particles $x = (X - X_0)$ distributions at the straw chamber planes (black histograms) in comparison with the TURTLE simulation results [9] (blue crosses), that were used for the simple beam model tuning. X_0 is the position of 75 GeV central beam particle.

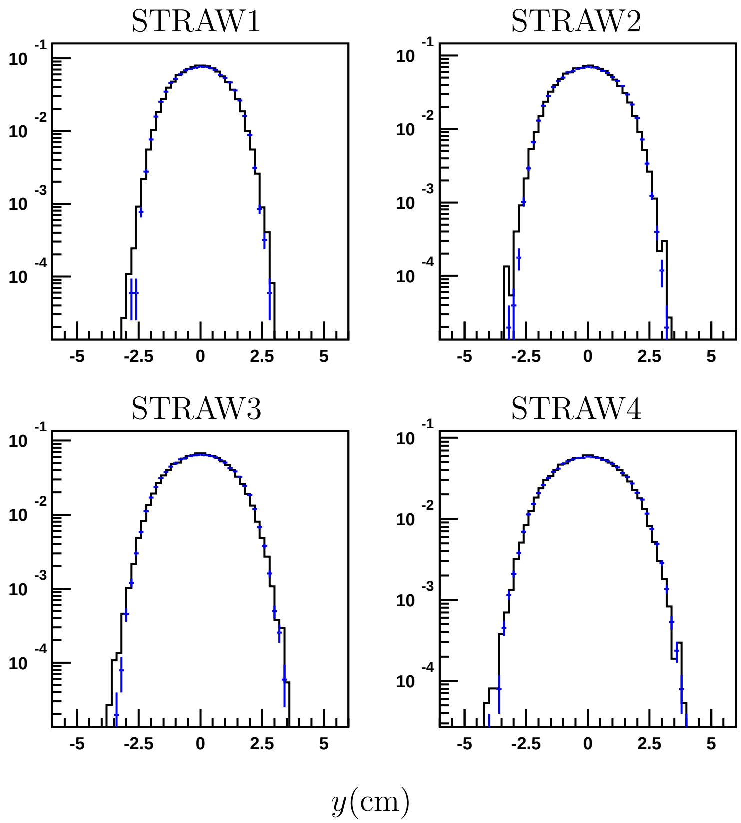

Figure 2: Normalized beam particles $y = (Y - Y_0)$ distributions at the straw chamber planes (black histograms) in comparison with the TURTLE simulation results [9] (blue crosses), that were used for the simple beam model tuning. Y_0 is the position of 75 GeV central beam particle.

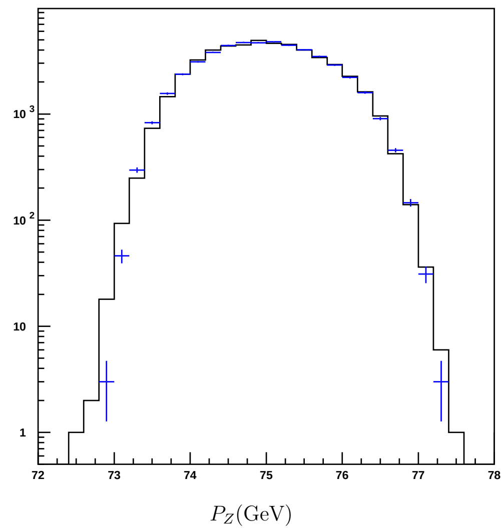

Figure 3: Histogram: momentum distribution of the simulated beam particles after the final collimator (at Z=102.4 m). Blue crosses - TURTLE simulation results [9], that were used for the present model tuning.

Figures 5, 6 show the distributions of X-view hits. Hit position is defined as the track point, that is closest to the wire of the straw.

3 Expected amount of restorable $K^+ \to \pi\nu A_{\pi\mu}$ events.

Kaon decas, apart from $\pi\mu$ atom production, were simulated using the corresponding decay codes from NA62MC [6], with the radiative corrections switched off (only Coulomb correction for $K^\pm \to 3\pi^\pm$ decay is taken into account).

$A_{\pi\mu}$ production in $K_{\mu 4}$ decays was simulated according to the matrix element expression (2). The decay probability distribution in E_ν, E_{π^+} plane is shown on the Fig. 7 (flat distribution over the phase space would correspond to the flat distribution in this two-dimensional space). To estimate the expected amount of detected $A_{\pi\mu}$, we first have simulated the main $\pi\nu\nu$ mode detection. For $\pi\nu\nu$ we use the selection conditions, related to the STRAW sub-detector, defined on the basis of true simulated physical values: fiducial region 10500.0 cm $< Z_{decay} <$ 16500.0 cm, missing mass $MM^2 < (2M^2_{\pi^0})$ and $MM^2 < 0.01\ (GeV/c^2)^2$ or $MM^2 > 0.026\ (GeV/c^2)^2$ (to exclude $\pi^+\pi^0$ area), positive pion momentum is between 15 and 35 GeV/c.

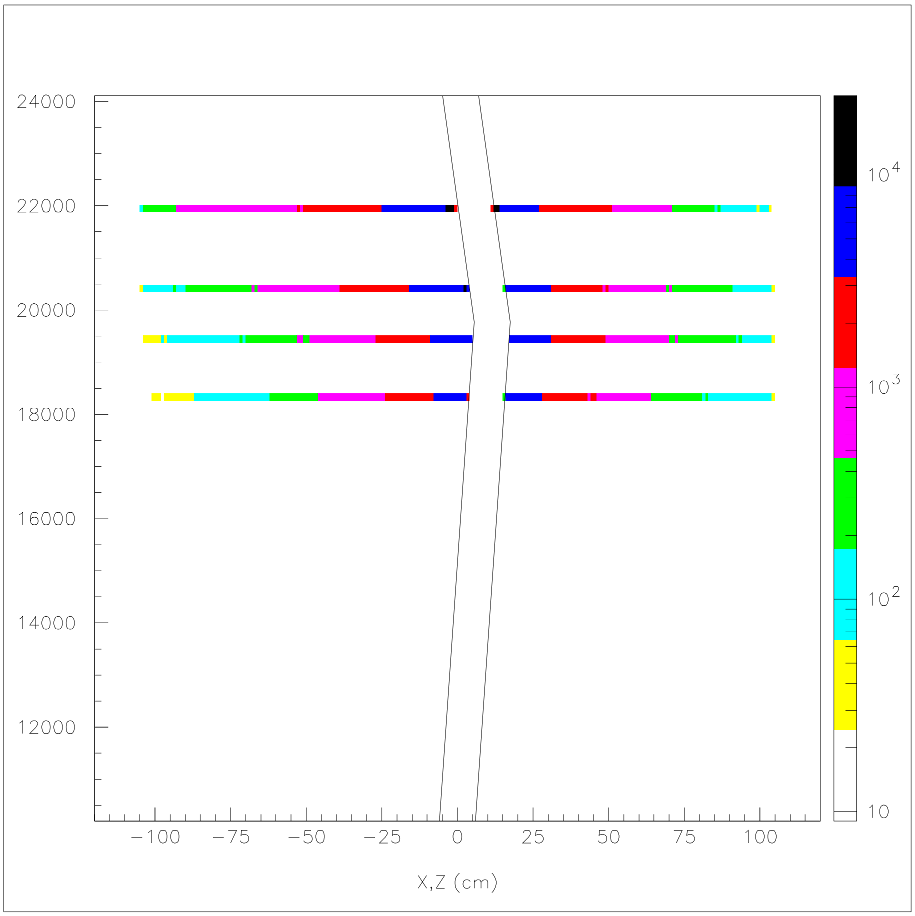

Figure 4: X-view straw hits (X,Z) distribution for 100000 random events, shown together with the lines, shifted by ±6 cm from the central 75 GeV particle path.

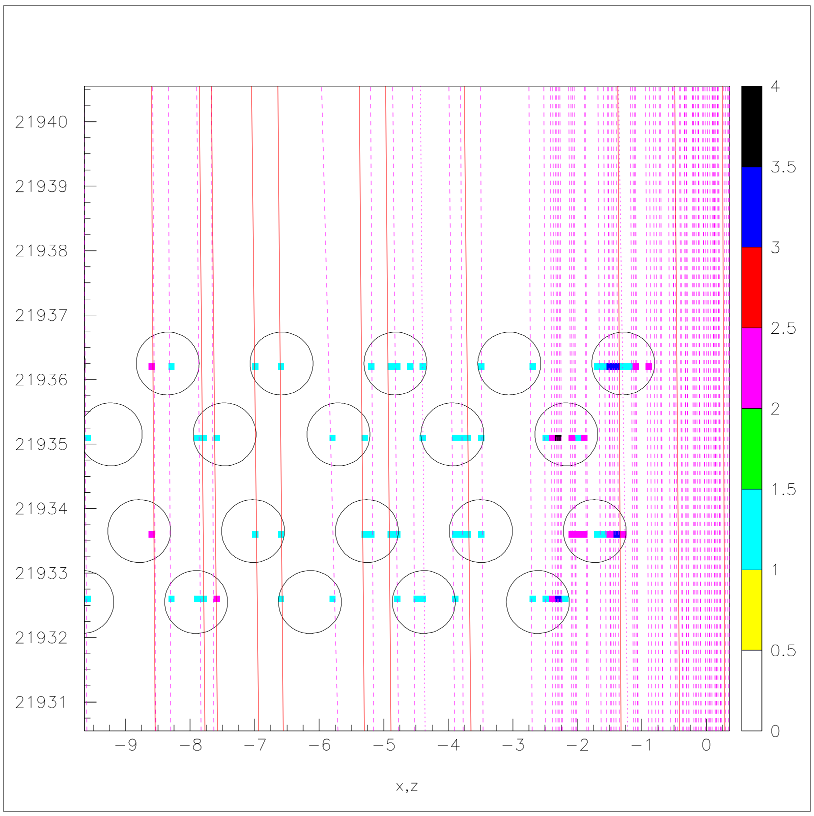

Figure 5: Tracks and X-view straw hits at the 4-th chamber near the central hole for 100 randomly triggered events. Red lines - positive pions, purple dashed lines - positive muons, purple dotted lines - positrons. Hits are plotted on the two-dimensional histogram.

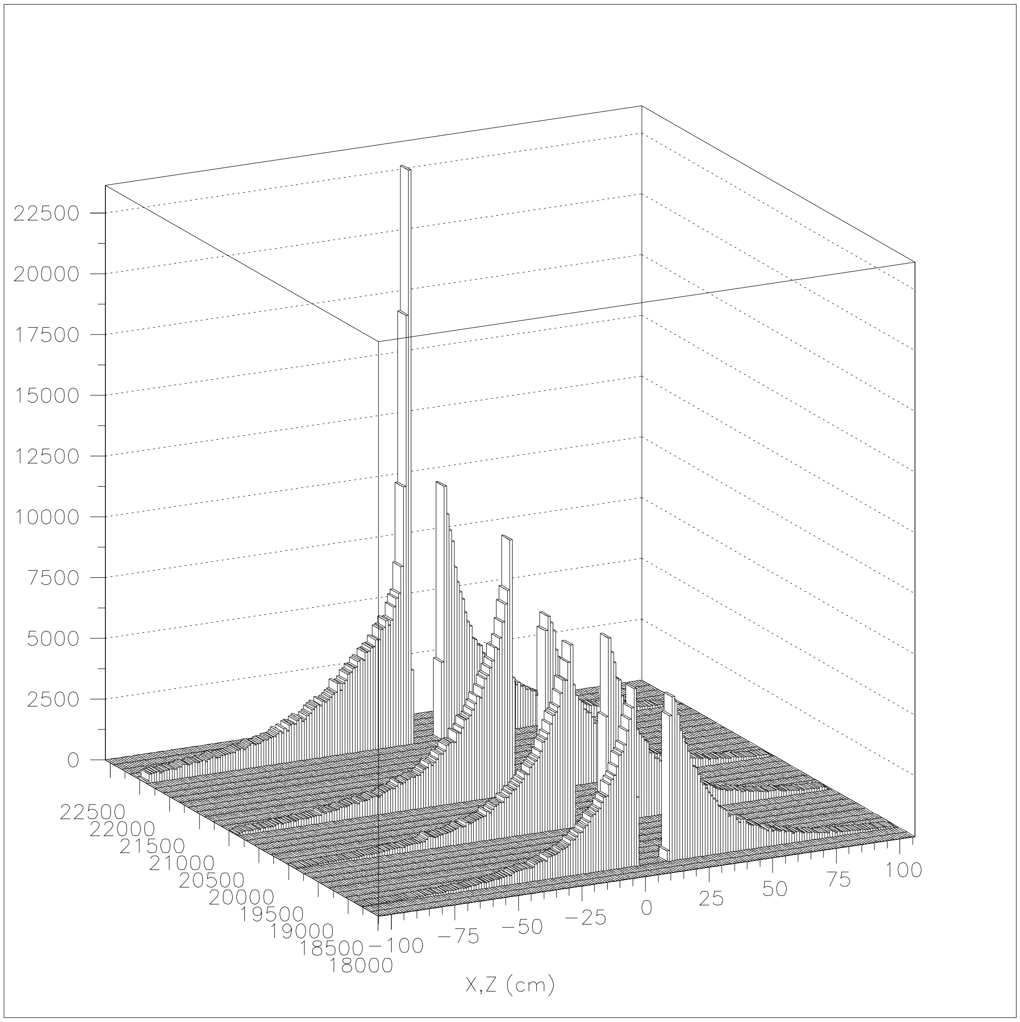

Figure 6: X-view straw hits (X,Z) distribution for 100000 random events, simulated as the mixture of 6 largest kaon decay modes.

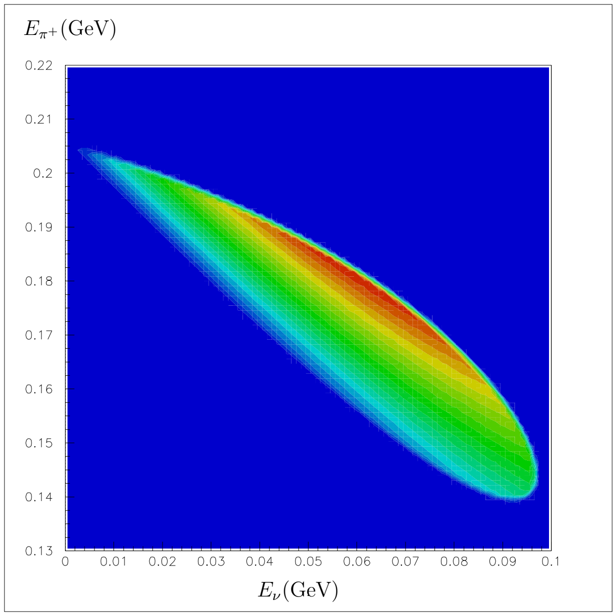

Figure 7: (E_ν, E_{π^+})-distribution of $\pi\mu$ atoms, produced in $K_{\mu 4}$ decay: simulation on the basis of matrix element expression (2).

Instead of reconstruction, the track is regarded as restorable if it has caused at least one hit in two different projections of every STRAW chamber. This condition may be somewhat optimistic in some special cases, but for the majority of events it just reflects the general geometrical acceptance condition for the track on the STRAW planes.

In order to take into account the possibility to reject the cases, when $\pi\mu$ atom decays upstream the last STRAW chamber and causes the extra hits, we implement the additional veto condition. Maximum allowed number of hits per every chamber was 11, and the same limit was set for the overall number of hits per every view (X,Y,U,V) of the complete STRAW detector.

To diminish the statistical fluctuations, we have simulated the events amount, that is an order of magnitude larger than the actually expected in the experiment. We have simulated 5000 $K^+ \to \pi\nu\bar{\nu}$ decays in the fiducial region and after applying of the above conditions 1056 events have been selected as restorable ones (see Fig. 9). It is about 10 times more than we plan to extract from NA62 experimental data.

Than we have simulated the corresponding expected amount of $K^+ \to \pi\nu A_{\pi\mu}$ decays in the same fiducial volume (according to the branching ratio between these two modes it is 2940).

Neutral $A_{\pi\mu}$ is undetectable for the straws until it decays (see Fig. 8) or break-up on the straw matter.

Break-up (or ionization) probability for the single straw wall is estimated as $P_{br}(\pi\mu) = 1 - exp(-\frac{\sigma_{ion}\rho N_A}{A}d)$, where $N_A = 6,022 \times 10^{23}$ is the Avogadro constant, $A = 12$ is the molar mass, equal to the carbon atomic number, $\rho = 14\ g/cm^3$ is the straw material density. In the case of multi-carbon molecule, the cross section is approximately proportional to the molar mass, so the formula remains valid. For the ionization cross section in carbon σ_{ion} we take the value of $1.37 \times 10^{-22} cm^2$, calculated in [11]. Taking the straw wall thickness of $d = 0.0036\ cm$, one obtains: $P_{br}(\pi\mu) = 0.034$[1]. Taking into account the typical amount of hits per direct track (≈ 10 hits in every chamber, two straw walls per hit) one can predict, that an essential fraction (≈ 0.5) of $A_{\pi\mu}$ atoms will be dissociated consequently on each straw chamber. But due to the technical limitations of the toy model this effect was not simulated here.

$A_{\pi^-\mu^+}$ decay is simulated in the assumption that π and μ are at rest in the atom rest frame. π and μ momenta in the lab frame are calculated as the fractions of $A_{\pi\mu}$ momentum, proportional to their masses. $\pi^- \to \mu^-\nu$ decay inside the atom is sampled according to the pion energy and mass. The final state charged muons interaction is not taken into account. The resulting μ^- together with the inner atomic μ^+ (that conserves its momentum after decay) are passed further and may cause the additional hits in the STRAW detector.

All $K^+ \to \pi\nu\bar{\nu}$ selection conditions are applied to these events apart from the missing mass cut, that would reject the majority of $K^+ \to \pi\nu A_{\pi\mu}$ due to the minimum missing mass of $MM^2 > (m_\pi + m_\mu)^2$ in this case. As a result, 1361 restorable events have been selected and their missing mass distribution is also shown on the Fig. 9. We have checked, that overlays with accidental events change the multiplicity veto response and suppress slightly both these signals depending on the accidental rate and veto time window. But it does not change essentially the ratio between the two signals.

In many cases $A_{\pi\mu}$ will be detected by STRAW after break-up or decay. In LKr and in the iron walls of MUV the hadronic part of the atom will interact with the matter nuclei and initiate the hadron and electromagnetic cascades, while the inner muon of the atom will mainly conserve its nature and will reach the MUV scintillator. So $K^+ \to \pi\nu A_{\pi\mu}$ have to be quite distinguishable from $K^+ \to \pi\nu\nu$ decays in NA62 setup even without taking

[1] In the Dirac experiment [12] at CERN, the production and break-up of $\pi^+\pi^-$ atom takes place in the target Ni foil. Taking the half of the foil with the thickness of $d = 0.0094 cm$ and with the cross section of $\sigma_{ion} = 1.8 \times 10^{-21} cm^2$ [11] the break-up probability for the average remaining path in the **single** foil is $P_{br}(\pi^+\pi^-) = 1 - e^{-0.755} = 0.53$. Taking into account the charge exchange $\pi^+\pi^- \to \pi^0\pi^0$ in the target, Dirac collaboration cited somewhat smaller probability $P_{br}(\pi^+\pi^-) = 0.452$. The authors are indebted to L. Di Lella and L. Afanasyev for the discussions on this topic.

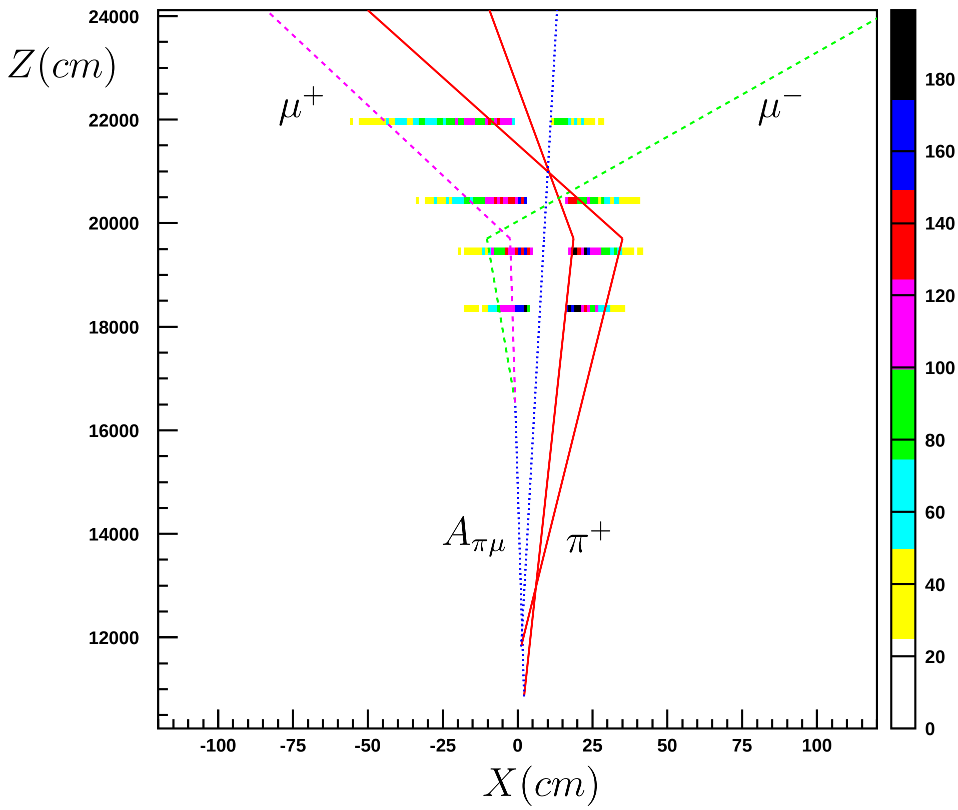

Figure 8: X-view straw hits (X,Z) distribution for 2940 $K^+ \to \pi\nu A_{\pi\mu}$ events (see text), shown together with two examples of this decay: one with the consequent $A_{\pi\mu} \to \mu^+\mu^-\nu$ decay and another one without it.

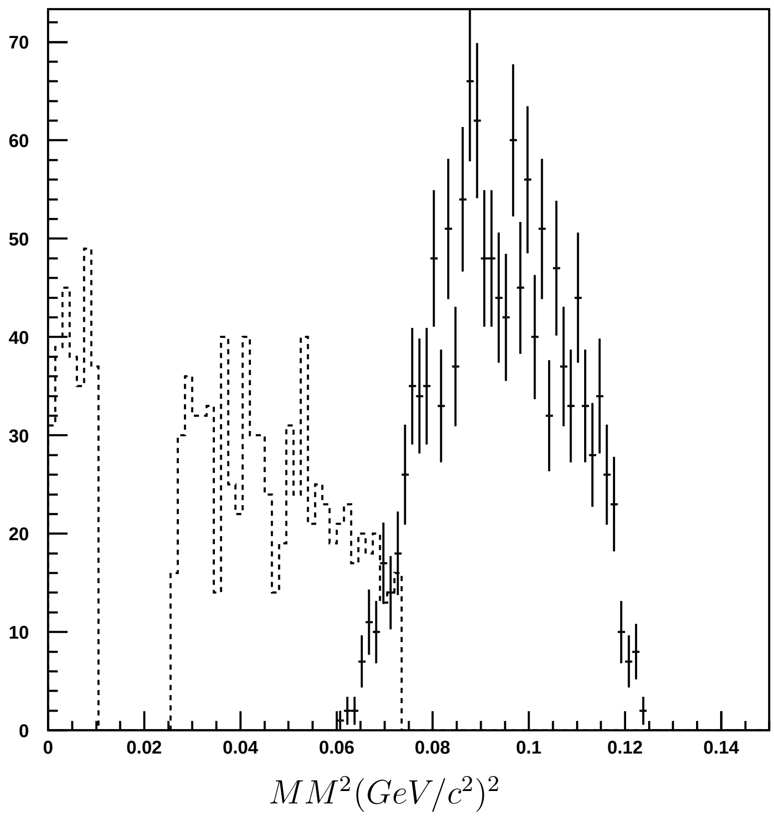

Figure 9: Distributions of missing mass MM^2 for restorable decays $K^+ \to \pi\nu A_{\pi\mu}$ (points) and $K^+ \to \pi\nu\nu$ (dashed histogram). Statistics is 10 times larger than one expected in experiment.

into account the difference in missing mass distributions.

Unfortunately for $K^+ \to \pi\nu A_{\pi\mu}$ decay, its registering may be completely suppressed at the trigger stage of NA62 experiment, as the muon veto and LKr will be used for the data rate decreasing. So to keep a possibility to see this decay, one have to foresee the considerable minimum bias sample. For example, the minimum bias sample, containing 20% of the complete triggered data, would provide about 28 restorable $K^+ \to \pi\nu A_{\pi\mu}$ events (apart from many other interesting rare decays).

4 Conclusion

The above consideration allows us to conclude that the NA62 experiment is able to detect in the minimum bias sample the amount of $K^+ \to \pi\nu A_{\pi\mu}$ decays, that is of the same order of magnitude as the $K^+ \to \pi\nu\nu$ signal (about 130 $K^+ \to \pi\nu A_{\pi\mu}$ per 100 $K^+ \to \pi\nu\bar{\nu}$). These events can not be a considerable contribution to the background for the main decay, due to the larger missing mass and additional response from STRAW, LKr and MUV. From the other hand, this elementary atom could be a suitable reference signal, that would show the ability of NA62 experiment to detect the decay with one reconstructable track and with the rate comparable to one of $K^+ \to \pi\nu\bar{\nu}$.

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