The future investigations of rare decays of the elementary particles demand the creation of new generation setups which can perform much more statistic and precise experimental data. Proposed new setup at CERN NA62 (NA48/3 (P326SPS)[1]) and at IHEP OKA [2] are planned to get experimental data on the level $10^{-10} - 10^{-12}$ branching ratio. Main goals of both experimental program connected with the study ultrarare kaon decays, but beams of these experiments have 95% (NA48) and 50% (OKA) pions. Naturally to use pion part of these beams for study rare pion decays. The pion program may be performed simultaneously or consecutive with main tasks. Such problems as search tensor interaction, measurements branching ratio and form-factors any pion decays, effects of polarization and search new particles are included in this program.

I. INTRODUCTION.

A new proposal P326 (CERN) [1] has a great goal: to measure ultra-rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. But the primary beam contains 95% of pions. In this note it is proposed to use of the pion part of beam for studying rare pion decays simultaneously with main task. The same program may be used in another setup OKA at U-70 accelerator (Protvino)[2]. For first setup main difficulties are connected with very high energy of particles of primary beam $\sim 75 GeV$. All secondary particles after decay are flying inside very narrow cone near axis of beam and passing dead space of detectors (Fig. 1).

But the momentum of secondary particles after decay is less than momentum of primary particles (Fig. 2) and they will deflected by spectrometer magnet M1 on large angles and their

*Electronic address: bolotov@inr.ru, bolotovvn@mail.ru
momentum and direction can be measured by second spectrometer M2 (Fig. 3). Secondary photons flying inside the holes of detectors will be registered by SAC at the end of setup.

For measuring of the photon momentum and space position the calorimeter must have high granularity and good energy resolution (Fig. 4).

The calorimeters with such parameters exist (See Table 1).

The parameters of the beam, acceptances and possible statistics for two main pion decays for P326 are listed in Table 2.

2. Experimental Methods for Study of Rare Decays.

There are two experimental methods of measuring the pion dacays: the former deals with stopped pions, the latter - with decays in flight. Typical layouts are shown on the Fig.5 the layout PIBETA for stopped pions and on the Fig.6 the ISTRA-M setup for pion decays in flight. ISTRA-M and PIBETA setups had been described in details in works [3] and [4].

The experiment in flight has some advantages compared with stopped pion one. Owing to the high detection efficiency, the wide range of measured angles and energies of secondary particles, and the substantial suppression of the background from the inelastic interactions in the target and from cascade

\[ \pi \rightarrow \mu \nu \quad \text{and} \quad \pi \rightarrow \mu \nu \gamma \]

\[ \rightarrow e \nu \gamma \quad \rightarrow e \nu \nu \]

takes place.

The experiments in flight allow to measure the energy and angle distributions in wide kinematical region. Experimental data in this case are more informative and reliable. For example, in Fig.7 kinematical regions for decay $\pi \rightarrow e \nu \gamma$ are shown, which could be investigated at stopped pions and in flight. The P326 experiment has all the advantages in flight methodic. It allows to get in the future very important and reliable experimental results.

In Table 3 rare decays of the charged pions are given, reflecting possible of the main routs of investigations.
3. Pion rare decays.

Radiative decay $\pi \to e\nu\gamma$ has been investigated in some works [4],[5],[6],[7],[11],[12]. The main subject of studying was the definition of axial-vector $F_A$ and vector form factors $F_V$. The amplitude of the radiative $\pi \to e\nu\gamma$ (Fig.8) decay is traditionally described in two terms corresponding to the inner bremsstrahlung (IB) and structure-dependent (SD) radiation. (Fig. 7) The IB contribution is closely connected with the $\pi \to e\nu$ decay and calculated by using the standard QED methods. The SD term is parameterized by two form factors $(F_V, F_A)$ that describe the vector ($F_V$) and the axial-vector ($F_A$) weak currents. The matrix element terms of decay (1) are given by:

$$M_{IB} = -ieG_F V_{ud} \sqrt{2} f_\pi m_e \varepsilon^\mu \varepsilon[(k/kq - p/pq)\mu + + \frac{\sigma_{\mu\nu} q^\nu}{2kq}] (1 + \gamma^5) \nu_e$$

$$M_{SD} = \frac{eG_F V_{ud}}{\sqrt{2M_\pi}} \varepsilon^{\mu\nu}[F_V e_{\mu\nu\rho\sigma} p^\rho q^\sigma + + iF_A (pq g_{\mu\nu} - p_{\mu} g_{\nu})] e_\gamma^\nu (1 + \gamma^5) \nu_e,$$

where $V_{ud}$ — CKM matrix element, $f_\pi = 131$ MeV- pion decay constant, $\varepsilon^\mu$-photon polarization vector, $p, k, q$ — 4-momenta of pion, electron and photon; $F_V$ and $F_A$ are vector and axial-vector form factors: $F_{V,A}(t) = F_{V,A}(0)[1 + \Lambda_{V,A} t/m_\pi^2]$. The consideration of QCD interactions of $\rho$ and $a_1$ mesons allows to calculate $\Lambda_V = m_\pi^2/m_\rho^2 = 0.033$ and $\Lambda_A = m_\pi^2/m_{a_1}^2 = 0.017$, hence it is possible to treat $F_{V,A}$ independent from $t$. Accordingly to CVC, $F_V$ is defined by $\pi^0$ life time:

$$|F_V| = 1/\alpha \sqrt{\frac{2}{2\pi m_{\pi^0} T_{\pi^0}}} = 0.0259 \pm 0.0005$$

The value $F_A$ depends on the model and ranges in a wide region from $-3F_V$ to $1.4F_V$ [2],[4],[5]. Usually ratio $\gamma = F_A/F_V$ is considered. The following kinematical variables are used: $x = 2E_\gamma/m_\pi, y = 2E_e/m_\pi$ and $z = (1 - y)/x$. It is also convenient to use variable $\lambda = (x + y - 1)/x = y\sin^2(\theta_{e\gamma}/2)$.

The differential probability of $\pi \to e\nu\gamma$ decay is given by

$$\frac{dW_{\pi \to e\nu\gamma}}{dx dy} = \frac{\alpha W_{\pi \to e\nu}}{2\pi} [IB(x, y) + \left(\frac{F_V m_\pi^2}{2 f_\pi m_e}\right)^2 \times (1 + \gamma)^2 SD^+(x, y) + (1 - \gamma)^2 SD^-(x, y)]$$

(3)
where IB and $SD^\pm$ are known functions:

$$IB(x, y) = \frac{(1 - y)[(1 - x)^2 + 1]}{x^2(x + y - 1)};$$  \hspace{1cm} (4)

$$SD^+(x, y) = (1 - x)^2(x + y - 1);$$

$$SD^-(x, y) = (1 - x)^2(1 - y).$$

The first studies did not give possibility to divide two values $\gamma$ [10],[11](See Table 4). With the starting of meson factories and the in-flight experiment the problem was settled. But values of this parameter in mostly statistics provided works differ more than two standard deviations [4],[5](See Table 4). Both these works were performed at stopped pions in narrow region of kinematics variables (angles between decaying e and $\gamma$ is about 180°). In work [7] investigations were done at angles region $0 - 180^0$. So reliability of definition $\gamma$ was very high. Probability of decay at this was on three standard deviations less than from Standard Model of V-A weak interactions. Investigated effect could be explained enormously big tensor interaction, destructively interfering with electromagnet interaction decreases possibility of decay. In works [7] and [13] analysis of existing experimental dates with tensor interactions was done. In Table 4 the results of this analysis are shown. It can be seen that taking into account the interference all values are in good agreement within errors. Theoretical works exist in which the antisimmetrical tensor fields are regarded [14]. In range of simple quark model (Fig.8) matrix element with tensor interaction could be written:

$$M_{\pi \rightarrow e\nu\gamma} = M_{IB} + M_{SD} + M_T;$$  \hspace{1cm} (5)

can be simulated by adding tensor radiation term to the structure dependent amplitude:

$$M_T = i(eG_FV_{ud}/\sqrt{2})\varepsilon^\mu q^\nu F_T u(p_e)\sigma_{\mu\nu}(1 + \gamma^5)\nu(p_\nu)$$  \hspace{1cm} (6)

The decay rate densities for the $SD^-$ radiation and the interference term between the inner bremsstrahlung and the tensor radiation are similar, so destructive interference may reproduce the results of fit, giving $F_T = -(5.6 \pm 1.7) \cdot 10^{-3}$. This value does not contradict the listed constraints on a tensor coupling from nuclear beta decay as well as from muon decay (if universality is supposed). This result does not contradict the previous experiments carried out with stopped pions either [13].

Several works [14] were devoted to the study of possible deviation from SM in radiative pion decay. In one of them the involving of antisymmetric tensor fields into the standard
electroweak theory allows to explain results of this work. It is evident that additional experimental and theoretical investigation of this problem should be carried out. In work [15] the authors show that there is region of phase space at large photon energies where the main physical background from muon decay is absent and that is optimal for searching a tensor interaction. In the analysis, it is convenient to describe the differential branching ratio as a function of the photon energy \( x = 2E_\gamma/m_\pi \) and the variable \( \lambda = (x + y - 1)/x = y \sin^2(\Theta_{e\gamma}/2) \). Formula (3) with tensor term gives

\[
\frac{dW_{\pi\rightarrow e\nu\gamma}}{dx\,dy} = \frac{\alpha W_{\pi\rightarrow e\nu}}{2\pi} \left\{ IB(x, y) + \left( \frac{F_V m_\pi^2}{2f_\pi m_e} \right)^2 \right\} \times
\]
\[
\times \{ [(1 + \gamma)^2 SD^+(x, y) + (1 - \gamma)^2 SD^-(x, y)] \} + \frac{F_T^2}{f_\pi m_e} T_1(x, \lambda) + \left( \frac{F_T^2}{f_\pi m_e} \right)^2 T_2(x, \lambda). \]

Here all terms with \( x \) and \( \lambda \) are factorized (see Fig.9);

\[
IB(x, y) = \frac{(1 - \lambda)(1 - x)^2 + 1}{\lambda x}; \]
\[
SD^+(x, \lambda) = \lambda^2 x^3(1 - x); \]
\[
SD^-(x, \lambda) = (1 - \lambda)^2 x^3(1 - x); \]
\[
T_1(x, \lambda) = (1 - \lambda)x; \]
\[
T_2 = \lambda(1 - \lambda)x^3. \]

From Fig.9 one could see that the most suitable region for searching a tensor interaction is \( 0.9 \leq x \leq 1.0 \) and that large opening angles between electron and photon (the experiments with stopped pions) correspond to the suppression of tensor interaction.

In work [16] the possibilities of the study of the tensor interaction at ISTRA setup were investigated. Fig.10 illustrates the distribution of a kinematical variable \( \lambda \) in the case of the existence of tensor interaction and without it.

In Fig. 11 given: results of decay \( \pi \rightarrow e\nu\gamma \) events simulated by Monte-Carlo method received at setup OKA in high energy region of variable \( 0.9 \leq x = 2E/m \leq 1.0 \). It can be seen, that even at low statistics of events existence of tensor interaction could be defined.
Recently a report on preliminary results performed by the PIBETA Collaboration from PSI meson factory has appeared [4]. The fits were made in two-dimensional kinematic space of 
\[ x = 2E_\gamma/m_\pi \] and \( \lambda = (x + y - 1)/x = y\sin^2(\Theta_{e\gamma}/2) \) on a very large statistical material (60k events \( \pi \to e\nu\gamma \) decays) (Fig.12). Fitting experimental data requires \( F_T \neq 0 \) and \( F_T \approx -0.0017 \pm 0.0001 \). Here is the quotation from PIBETA Annual Progress Report:

”Thus, like the ISTRA data, our data appear to call for a destructive interference between the IB term and a small negative tensor amplitude.” (Fig.12)

The results from ISTRA and PIBETA setups don’t solve the problem. That study should be continued.

In work [17] an interesting consequence for radiative decay \( \pi \to e\nu\gamma \) from hypothesis electromagnetic neutrino momentum (DM) is investigated. If neutrino of Majorana fermion type exists, the amplitude of dipole momentum interfering with structure depended part can cause distortion of energetic spectra of the secondary electrons and photons (Fig.13). Assuming the neutrinos have masses and electric and magnetic dipole moments DM the addinional terms to (2), (3) of Standard Model amplitudes appear:

\[
|M_{\pi\to e\nu\gamma}|^2 = |M_{IB}|^2 + |M_{CD}|^2 + |M_{DM}|^2 + 2Re(M_{IB}^+M_{DM}) + 2Re(M_{SM}^+M_{DM}),
\]

The conditions \( Im(\mu_{ji} + i\bar{d}_{ji}) \neq 0 \) and \( \Delta m_{ji}^2 = m_i^2 - m_j^2 \) may fulfill only for Majorana neutrinos.

In this case the outcome photon can be attached to the neutrino leg (Fig.13). Imaginary part of decay amplitude (Fig.14) also may be nonzero if \( Im(\mu_{ji} + i\bar{d}_{ji}) \neq 0 \)

Hitherto the experimental limits on magnetic and the electric DM are [18],[19]:

\[
|\mu_{\nu(e)} + i\bar{d}_{\nu(e)}| < 1.5 \cdot 10^{-10} \mu_B, \quad |\mu_{\nu(\mu)} + i\bar{d}_{\nu(\mu)}| < 1.2 \cdot 10^{-9} \mu_B, \quad |\mu_{\nu(\tau)} + i\bar{d}_{\nu(\tau)}| < 4 \cdot 10^{-6} \mu_B,
\]

where \( \mu_B = e^2/2m_e \) is Bohr magneton. The distributions from [17](see Fig.15) are obtained assuming experimental limits from \( \pi \to e\nu_j \) decay [20], a mixing matrix element \( U_{e3} = 10^{-2} \), a neutrino \( m_3 = 5\text{MeV} \) and a neutrino magnetic DM \( \mu_{13} = 4 \cdot 10^{-6} \mu_B \). To improve experimental limit on \( \mu_{13} \) it is necessary to get the level of branching ratio of \( 10^{-12} \). It is difficult but possible.

Probability of these decays is very low < \((1.4 - 5.1)10^{-15}\).
Checking T-invariance. The decays $\pi \to l\nu\gamma$, where $l = \mu, e$, may serve for testing T-violating interactions beyond the Standard Model. $\pi \to e\nu\gamma$ and $\pi \to e\nu e^+ e^-$ must be real. In case of violation T-invariance they have imaginary part. At maximum of violation the value of imaginary part must be equal to real part. In this case T-odd correlations must appear. In case $\pi \to e\nu\gamma$ decay they are expressed by polarization vectors of positron ($\sigma$) and photon ($\epsilon$) : $\sigma(\overline{k} \times \overline{p})$ (A) $\epsilon(\overline{k} \times \overline{p})$ (B). Correlation (A) is expressed only via transverse positron polarization $P_T$. In some spaces of phase volume $P_T$ can get 67% [18].

In work [21] (for these decays it was investigated) the possibilities of measuring the transverse lepton polarization asymmetry in these decays were investigated:

$$P_T(x, y) = \frac{d\Gamma(\overline{e}_T) - d\Gamma(-\overline{e}_T)}{d\Gamma(\overline{e}_T) + d\Gamma(-\overline{e}_T)},$$

where polarization in direction $\overline{e}_T = \frac{[\overline{k} \times q]}{|\overline{k} \times q|}$ and $k, q$ - are 4-momenta of lepton and photon.

In figures 16 and 17 the distributions of differential branching ratio of $\pi \to \mu\nu\gamma$ and $\pi \to e\nu\gamma$ decays over the Dalitz plot and the transverse lepton polarization asymmetry $P_T$ as background from Standard Model interactions are shown. It can be seen, that regions with large $P_T$ overlap with regions of large branching ratio of decays.

The transverse lepton polarization asymmetry $P_T$ can’t be measured in flight experiments because the lepton energies are too high. It is possible to study this polarization in $\Gamma_g$ and $\Gamma_h$ contributions to the total polarization as function of photon energy (see Fig.18)[22].

Decay $\pi \to e\nu e^+ e^-$ is connected with decay $\pi \to e\nu\gamma$ (see Fig.19).

Besides known V and A form factors this decay is described by another form factor $R$, connected with electromagnetic radius of pion. Matrix elements $M$ for SD in vector formfactor $F_V$ and axial form factors $F_A$ and $R$:

$$M(SD_V) = \frac{-eG}{\sqrt{2}m_\pi} \varepsilon^\mu l^\nu F_V \varepsilon_{\mu\nu\sigma\tau} k^\sigma k^\tau,$$

$$M(SD_A) = \frac{-ieG}{\sqrt{2}m_\pi} \varepsilon^\mu l^\nu \{F_V [(s - t)g_{\mu\nu} - q_\mu k_\nu] + R t g_{\mu\nu}\},$$

where $\varepsilon^\mu$ - vector polarization of real photon or electron-positron current, $\varepsilon^\mu = (e/t)\overline{u}(p_-)\gamma^\mu v(p_+), l^\nu = \overline{u}(p_\nu)\gamma^\nu (1 - \gamma_5)(p_\nu)$, $q$ and $k$ - fourvector of pion and photon and $s = qk, t = k^2$. 
If Conserving Vector Current (CVC) hypothesis is fair then $F_V$ is connected with life time of $p^0$ meson: $|F_V| = (1/\alpha)(2\pi/\pi m_{\pi})^{1/2}$. Hypothesis of partly conserving axial vector current (PCAC) connects $R$ with electromagnetic pion radius:

$$R = (1/3)m_{\pi}f_{\pi} \ll r_{\pi}^2 \gg.$$ To radius $< r_{\pi}^2 > = (0.432 \pm 0.024)$ corresponds value $R_{PCAC} = (0.068 \pm 0.004)$. In work [9] at statistics of 98 events values $\Gamma(\pi \rightarrow e\nu e^+ e^-)/\Gamma(\pi \rightarrow \mu\nu) = (3.2 \pm 0.5) \times 10^{-9}$ and $R = 0.063$.

**Anomalous interactions.** Decay $\pi \rightarrow e\nu e^+ e^-$ gives possibilities for searching anomalous 4- and 6-fermion interactions (see Fig.20).

Probability of these decays is very low $< (1.4 - 5.1) \times 10^{-15}$.

Decay $\pi \rightarrow \mu\nu\gamma$. For studying structural matrix element this decay is not interesting because CD terms are strictly depressed by internal bremsstrahlung(IB) in total region of phase volume.

In some works increased number of events was noticed in low energy part of photon specet[24]. The some effect was noticed in work [25]. Up to 1998 this was the only work devoted to this decay. In 1998 at small statistics (about 300 events) this problem was investigated in work [26]. Deviation from prediction of quant electrodynamics was not observed. Threshold of photons registration was 1 MeV. Branching ratio of the decay was got: $BR(E_{\gamma} > 1MeV)_{exp} = 2.0 \times 10^{-4} \pm 12\%(stat) \pm 4\%(sys)$.

Some interest to this decay may be connected with studying the transverse lepton polarization (see Fig.16,17) and work[21].

Decays $\pi \rightarrow evM$. $M \rightarrow (m, h)$ can be new scalar, pseudo scalar, vector or pseudo vector particle with mass $0 \leq M \geq 130MeV$. These decays give more information which can be received during investigation of other elementary particle decays. For example, $K \rightarrow \pi M$, where $M$ can be only scalar or pseudo scalar, or $\mu \rightarrow eM$ lepto number violation.

They connect possible existence of particle $M^0 \rightarrow (m, X)$ with hypothetical models. For example, maioron neutrino or light neutral bosons(see Fig.21)[27][26]. In work [28] the differential width of Higgs boson $h$ and in work [29] a branching ratio $B(h \rightarrow e+e-) > 0.9$ are given.

$$B_{th}(\pi \rightarrow e\nu h) = [\sqrt{2}G_F m_{\pi}^4 f(x)]/[48\pi^2 m_\mu(1 - m_\mu^2/m_{\pi}^2)^2],$$

where $f(x) = (1 - 8x + x^2)(1 - x^2) - 12x^2 \ln x$ and $x = m_h^2/m_{\pi}^2$. On the Fig.22) the
upper experimental limit and prediction for branching ratio of $\pi \rightarrow e\nu\bar{h}$ are given [30].

ACKNOWLEDGMENTS

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[1] Proposal to Measure the Rare Decay $K^+ \rightarrow \pi^+\nu\nu$ at the CERN SPS. Preprint CERN-SPSC-P-326-013, SPSC-P-326, 116(2005).
[16] V.N. Bolotov et al.,Preprint IHEP,95-111,Protvino,(1995);
   A. Belogianni et al. CERN PPE/96-145(1996);
Fig.1. Angles between primary pions and secondary electrons without and with taking into account detector resolution.

Fig.2. The momentum distributions for electrons from $\pi^+ \rightarrow e^+ \nu \gamma$ decay for $\gamma_c = 537(p_\pi = 75 GeV/c$, red line) and $\gamma_c = 175(p_\pi = 25 GeV/c$, blue line).

Fig.3. Radiative pion decay at P326.

Fig.4. SHASHLIK tower.

Fig.5. The layout of PIBETA setup.

Fig.6. The layout of ISTRA-M setup: S1- S5 - scintillation counters; C1-C4 - Cherenkov gas counter; M1, M2 - beam and spectrometer magnets; PC1-PC6 - proportional chambers; DC1-DC16 - drift chambers; EC1 and EC2 - lead glass electromagnetic calorimeters; DT1-DT8 - drift tubes; MH - matrix hodoscope; HC - hadron calorimeter; MD - muon detector.

Fig.7. The kinematic regions of the $\pi \rightarrow e\nu\gamma$ decay at the ISTRA setup (gray) and in the stopped pion experiments (black). Black points correspond to the maximum values of the $SD^\pm$ terms.

Fig.8. The contributions to the radiative decay $\pi \rightarrow e\nu\gamma$ in the framework of quark model: a) and b) contain IB; c) and d)- SD.

Fig.9. The dependences of the differential branching ratio as a function of the photon energy $x$.

Fig.10. The distribution of the kinematical variable $\lambda$ theoretical (a) and with experimental errors taken into account (b). Dashed line corresponds to the existence of the tensor interaction; solid line to absence of this interaction.

Fig.11. Distribution of radiative decay $\pi \rightarrow e\nu\gamma$ events over variable $\lambda$. Events are chosen for other variable $x$ in high energy region of decaying $\gamma(0.9 < x < 1.0)$. Calculations were performed by Monte-Carlo method for one variant OKA setup.

Fig.12. Measured spectrum of the kinematical variables $\lambda$ in $\pi \rightarrow e\nu\gamma$ decay. Dotted curve: fit with $F_V$ fixed by CVC hypothesis, and $F_A$ taken from PDG 2002 compilation. Dashed curve: fit with $F_V$ constrained by CVC, and $F_A$ and $F_T$ unconstrained. The resulting value for $F_T$ is $-0.0017 \pm 0.0001$. Preliminary results - work in progress.

Fig.13. Feynman diagram for the process $\pi \rightarrow e\nu\gamma$ when the photon is emitted from the neutrino leg.

Fig.14. Loop diagram for the lowest-order contribution to the radiative pion decay through neutrino nondiagonal DM.
Fig. 15. a) Differential probability of radiative decay $d\Gamma/dx$, integrated in region $1 - 0.8x \leq y \leq 1 + r^2$, where $r = m_e/m_\pi$. Values $d\Gamma/dx$ are free. For interferential SD-IB term absolute values are given. b) the same as a) for $d\Gamma/dx$ in region $0.3 \leq x \leq 1 - r^2$.

Fig. 16. The distribution of differential branching ratio of $\pi_{\mu2\gamma}$ and $\pi_{e2\gamma}$ decays over the Dalitz plot.

Fig. 17. Transverse lepton polarization due to FSI in $\pi_{\nu2\gamma}$ and $\pi_{e2\gamma}$ decays.

Fig. 18. The absolute value of $\Gamma_i$ the contributions to the total decay rate $(i=a(SD),b(SD),c(IB), d \text{ and } e(INT))$ and to the total polarization $(i=g,h)$.

Fig. 19. Diagrams of structural (SD) and internal Bremsstrahlung (IB) interactions of decay $\pi \to e\nu e^+e^-$.

Fig. 20. "Anomalous" 4- and 6-lepton interactions.

Fig. 21. Diagram for Maioron or unknown radiations in $\pi \to e\nu M$.

Fig. 22. Prediction of the branching ratio $B_{th}(\pi \to e\nu h)$ and experimental upper limit (90%CL) $B_{exp}(\pi \to e\nu h)$ as a function of the Higgs mass $m_h$. Solid curves represent electron decays and dashed curves represent muon decays.

Figures
FIG. 14:

FIG. 15:

FIG. 16:
FIG. 20:

FIG. 21:

FIG. 22:

Tables
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<td>$K^+$ beam flux at production ($p_k = 75(GeV/c)$</td>
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<td>$\pi^+$ flux</td>
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<td>Decays</td>
<td>Branching ratio $R = \Gamma_i/\Gamma_{all}$</td>
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<tr>
<td>$\pi \rightarrow e\nu\gamma$</td>
<td>$(1.61 \pm 0.23) \times 10^{-7}$</td>
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</table>
\[ \gamma = \frac{F_A}{F_V} = 0.475 \pm 0.018 \]
\[ F_T = -(2.2 \pm 0.1) \times 10^{-3} \]
\[ \theta_{e\gamma} > 135^\circ \]

( PSI,03),[4]

2. \( \pi \to \mu \nu \gamma \) \( (2.0 \pm 0.24) \times 10^{-4} \)

Two works with low statistics [8]

Problem in the \( \gamma \) low-energy region.

3. \( \pi \to e \nu \pi^0 \) \( (1.038 \pm 0.004 \pm 0.007) \times 10^{-8} \)

Well studied.

Check of Stand.M.

\[ V_{ud} = 0.9737 \] [9]

PDG02 \[ V_{ud} = 0.9734 \]

4. \( \pi \to eee \) \( (3.2 \pm 0.5) \times 10^{-9} \)

\[ F_V = 0.023 \pm 0.014 \]
\[ F_A = 0.021 \pm 0.012 \]
\[ R = 0.059 \pm 0.008 \]

Study of form factors \( F \) and \( R \)

98 ev. [10]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \gamma(F_T = 0) )</th>
<th>( \gamma^*(F_T = -8.6 \times 10^{-3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN (P. Depommier et al.1963,[11])</td>
<td>0.26 \pm 0.15 (-1.98 \pm 0.15)</td>
<td>0.66 \pm 0.15</td>
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<tr>
<td>Berkeley(A.Stetz et al.1978([12]))</td>
<td>0.48 \pm 0.12 (-2.42 \pm 0.12)</td>
<td>0.60 \pm 0.12</td>
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<td>SIN (A.Bay et al.1986,[5])</td>
<td>0.52 \pm 0.06</td>
<td>0.63 \pm 0.06</td>
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<tr>
<td>LAMPF (L.Philonen et al.1986, [6])</td>
<td>0.22 \pm 0.15</td>
<td>0.48 \pm 0.15</td>
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<tr>
<td>INR (V.Bolotov et al.1990, [7])</td>
<td>0.41 \pm 0.23</td>
<td>0.41 \pm 0.23</td>
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</tbody>
</table>

TABLE IV: Measured and corrected values of \( \gamma \).